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Wind generator network methodology and analysis

James Neils Peterson
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WIND GENERATOR NETWORK METHODOLOGY AND ANALYSIS

Iowa State University

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Wind generator network methodology and analysis

by

James Neils Peterson

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Graduate Faculty in Partial Fulfillment of the
Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major: Electrical Engineering

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CHAPTER 1.

ELECTRICAL POWER GENERATION FROM THE WIND

Introduction

There is increasing interest nation-wide in wind-generated electrical power ^(1,2). The most visible evidence of this is the NASA/DOE program to experimentally operate several large wind generators. The first unit was constructed near the NASA/Lewis Laboratory at Plum Brook, Ohio in 1975 ^(3,4) which generates 100 kW at the rated wind speed of 18 mph. It has been used as a test bed for engineering designs and subsystem improvements. A 200 kW unit is now operating at Clayton, New Mexico, and a 2 megawatt wind generator has recently been installed at Boone, North Carolina. The wind generator at Boone is the largest ever built; the two blades span a diameter of 200 ft and the supporting tower is 140 ft tall ⁽⁵⁾. Other larger wind generators are in the planning/development stage. The Boeing Company has been awarded a \$10 million contract from DOE to produce a wind generator with a swept area of 300 ft dia and a power capability of about 2.5 megawatts.

A private company, Wind Power Products Co. of Seattle, which has recently been purchased by the Bendix Corporation, has developed a large-scale wind generator and sold an experimental model to Southern California Edison Company. The wind generator unit, to be located in a pass in the mountains inland from Los Angeles, is currently being assembled and will be used to gain operational experience as well as to produce some usable electrical power. The three-bladed wind generator is rated at over 2 megawatts in a 40 mph wind ⁽⁶⁾.

In addition to modern large-scale wind generators, such as mentioned above, small scale wind generators are again increasing in popularity (7,8). Such units, although potentially beneficial for specific individual needs, are not expected to have a major impact on the energy supply. As with other electrical generating systems, there is an economy of scale with wind generators. Also, the strong dependence of wind speeds on terrain features, coupled with the fact that wind power is proportional to the cube of the wind speed, suggests that individuals may not have access to good wind power sites.

Prior to the large scale production of electrical power from wind generators, many issues must be resolved (9). These include structural designs, control and protection equipment, multi-unit operations, interfacing with existing electric utility grids, and site selection. In this report, the site selection issue is addressed from the viewpoint of a wind generator network and methods are developed which can aid in the site selection process and in wind generator network analysis. Other studies which have examined various aspects of large-scale wind generators are summarized in the following.

Related Work

A two-year study recently completed by the General Electric Company for the Electrical Power Research Institute (EPRI) presents preliminary impact and penetration analyses of wind generation in three actual utility systems (10). Using economic evaluation techniques consistent with current use in the electric industry, the value of the wind power plants was determined in the three systems for different penetration levels.

The three utility systems used in the study are the Kansas Gas and Electric Company, the Niagara Mohawk Power Corporation (in northern New York) and the West Group of the Northwest Power Pool (in the Pacific Northwest). In an hour-by-hour simulation, one year of "typical site" wind speeds were used in various wind generator models to produce the expected wind power. Then in the planned electric utility generation system for 1990 (1995 for the Northwest), wind generator units were substituted for some of the planned conventional units. The total system costs for wind generator penetration levels from 0 to 20 percent were then compared.

The General Electric Company study uses what is termed a "typical year" of wind data for the wind generator simulation analysis. This typical year is the actual time series of wind speeds from a specific site with monthly and annual average wind speeds that appear to be representative of winds in the region. Using this approach allows various wind generator/conventional generation configurations to be analyzed from a common basis. The study does not examine the effects of geographical dispersions of wind generators, which could improve the effective capacity of the wind generators.

In a Bureau of Reclamation study, Hightower and Watts ⁽¹¹⁾ have examined the combination of multiple wind generators and a hydroelectric system. The proposed wind site is Medicine Bow, Wyoming, with standard electrical transmission facilities used to couple the wind generators with a hydroelectric system. A single site is considered for the wind generators, with the smoothing of fluctuations accomplished by coupling with the hydro storage and not by geographical dispersion. Annual wind speed

durations are considered in this preliminary study and not actual time histories of wind speeds and hydro storage levels. Another Bureau of Reclamation study ⁽¹²⁾ considers the problem of interconnecting the wind generators with hydroelectric storage using long distance electrical transmission and explores the economic aspects.

Analysis of wind speeds for potential power production can be accomplished using actual time histories of wind speeds or by using estimates of wind speed probability density functions. Actual wind speeds provide more accurate and detailed analysis, but such data may not be available for sites of interest. This has led some researchers to develop techniques for estimating the wind speed distributions from typical weather data, such as average wind speed and other basic wind speed statistics ^(13,14,15). This statistical approach has the convenience of describing wind speeds with only a few parameters, instead of the many data points for actual wind speeds. Only two parameters are needed to define the Weibull distribution, which seems to be an acceptable distribution for describing wind speeds ^(16,17). For analyzing many sites ^(18,19) the probability-distribution approach is computationally attractive. However, for specific sites in a potential wind generator network, and in regions of irregular terrain ⁽¹⁷⁾, wind speeds should be used directly (if available) in order to provide accurate cross-correlations as well as power densities. In this study actual hourly wind speeds are used since the data are available for the sites considered, the sites are in a region of highly irregular terrain (the Pacific Northwest), and the cross-correlations among the sites are crucial in determining the network power fluctuations.

A number of wind energy studies have been conducted, with the primary objective in most cases to determine an overview of the wind power potential. Reed ⁽²⁰⁾ has compiled various sources of wind data into a nationwide evaluation of average wind power densities. A similar nationwide analysis has been conducted by Justus et al. ⁽¹⁸⁾ using probability density functions. In the Pacific Northwest several studies of wind energy have been conducted ^(21,22,23) with the objective of identifying regions or sites with good wind power. One study ⁽²²⁾ also considers the integration of wind generators with existing hydroelectric facilities.

Overview

This study examines the power production of a wind generator network and the advantages of unequal allocation of wind generator units among the several sites comprising a wind generator network. Actual time histories of hourly wind speeds are used, rather than estimated probability distributions of wind speeds.

Chapter 2 presents a simplified analysis of a 3-site wind generator network. Optimal weightings are calculated for allocating the wind generators among the sites, using various assumed means and variances. The actual wind data used in this study is described in Chapter 3. Wind data sites are identified and wind power duration curves for each of the sites are shown, and the cut-in, rated, and cut-out wind speeds used for the study are described. Chapter 4 examines the network power production and fluctuations using actual wind speed data from several different sites. Optimal allocation weightings are computed and used, based on both annual and monthly wind speed characteristics. A summary of the results and the conclusions are presented in Chapter 5.

CHAPTER 2.

SITE SELECTION METHODOLOGY

Introduction

Large scale production of electrical power from wind generators grouped at several isolated sites will have fluctuations according to the variations at each site and the correlations among power production at separate sites. The approach to the selecting and sizing (i.e., determining number of wind generators at each site) of sites should include a consideration of the power fluctuations from the proposed wind generator network. By accounting for the variations at each site and the correlations from site to site, the percent of wind generators at each site can be determined so that the resulting network power fluctuations will be minimized while still achieving a desired level of electrical power production.

This can be stated as a combined minimization/maximization problem: minimize the combined power fluctuations while maximizing the total energy production. It is obvious that these two criteria tend to be opposing goals, so that a compromise is necessary. Two approaches will be examined: (1) for a fixed value of allowed network power fluctuations, choose the site weightings to maximize the energy production; (2) for a fixed value of energy production, choose the site weightings to minimize the network power fluctuations. It is likely that under certain conditions, these two approaches will yield identical results. The weightings, thus determined, will indicate the percentage of total windmills to be located at each site.

Optimal Weightings

To develop the site selection procedure, consider first the power available from a single site. Let x represent the instantaneous power from one wind generator and let w represent the number of wind generators to be located at this site. Then, the average power production and the variance of the fluctuations can be readily determined.

$$\text{total power} = wx \quad 2-1$$

$$\begin{aligned} \text{average power} &= E(xw) \\ &= wE(x) \\ &= w m_x \end{aligned} \quad 2-2$$

$$\text{variance} = w^2 \sigma_x^2, \quad 2-3$$

$$\text{where } \sigma_x^2 = E(x - m_x)^2$$

Now consider the case of power production from a wind generator network. Assume for the present that the fluctuations are uncorrelated from site to site.

$$\text{total power} = \sum_i w_i x_i \quad 2-4$$

$$\begin{aligned} \text{average power} &= E\left[\sum_i w_i x_i\right] \\ &= \sum_i w_i m_i \end{aligned} \quad 2-5$$

$$\text{variance} = E\left\{\left(\sum_i w_i x_i - \sum_i w_i m_i\right)^2\right\} \quad 2-6$$

and, if uncorrelated,

$$\text{variance} = \sum_i w_i^2 \sigma_i^2, \text{ where } \sigma_i^2 = E(x_i - m_i)^2 \quad 2-7$$

This variance of the network power production can be reduced from that of the single site (eq. 2-3), where

$$w = \sum_i w_i \quad 2-8$$

by choosing the values w_i appropriately. That is, it is always possible to choose the weightings $w_i \geq 0$ such that

$$w^2 \sigma_j^2 \geq \sum_i w_i^2 \sigma_i^2, \text{ where } w = \sum_i w_i \quad 2-9$$

This can be shown as follows: First choose the weighting vector \underline{w} to minimize the right hand side of 2-9. The value of \underline{w} that does this is determined as

$$\underline{w} = \frac{w}{\sum_i \frac{1}{\sigma_i^2}} \begin{bmatrix} \frac{1}{\sigma_1^2} \\ \vdots \\ \frac{1}{\sigma_n^2} \end{bmatrix} \quad 2-10$$

For this value of \underline{w} , the right side of 2-9 becomes

$$\begin{aligned} \sum_i w_i^2 \sigma_i^2 &= \frac{w^2}{\left(\sum_i \frac{1}{\sigma_i^2}\right)^2} \sum_i \frac{1}{\sigma_i^2} \\ &= \frac{w^2}{\sum_i \frac{1}{\sigma_i^2}} \end{aligned} \quad 2-11$$

Multiply numerator and denominator of the right hand side of eq. 2-11 by σ_j^2 ; the resulting expression can be written as

$$\sum_i w_i^2 \sigma_i^2 = \frac{w^2 \sigma_j^2}{1 + E} < w^2 \sigma_j^2 \quad 2-12$$

where

$$E = \sum_{i \neq j} \frac{\sigma_j^2}{\sigma_i^2} > 0 \quad 2-13$$

This proves inequality 2-9.

For the case of equal weightings and variances at all sites, this reduces to the familiar relationship

$$\sigma_{\text{network}}^2 = \frac{\sigma_{\text{site}}^2}{n}, \text{ where } n \text{ is the number of sites} \quad 2-14$$

This emphasizes the advantage of dispersing wind generators over a wide geographical area, which is to obtain a significant reduction in the network fluctuation of wind generated power.

The wind power available at different sites, however, will have different mean values, different variances, and there will likely be some degree of correlation in the variations of power from site to site. This raises the question of whether there is an optimal method of allocating wind generators among various sites so as to maximize the average network power production while minimizing the network power fluctuations. The following considers the general case of n sites with different statistics at different sites.

Let the total number of wind generators be w and the number at the i th site be w_i . Then

$$w = \sum_i w_i \quad 2-15$$

and

$$\text{network power} = \sum_i w_i x_i = \underline{w}^T \underline{x} \quad 2-16$$

$$\text{average power} = \sum_i w_i m_i = \underline{w}^T \underline{m} \quad 2-17$$

$$\begin{aligned} \text{network variance} &= E(\underline{w}^T \underline{x} - \underline{w}^T \underline{m})^2 \\ &= \underline{w}^T E[(\underline{x} - \underline{m})(\underline{x} - \underline{m})^T] \underline{w} \\ &= \underline{w}^T P \underline{w}, \text{ where } P = E[(\underline{x} - \underline{m})(\underline{x} - \underline{m})^T] \quad 2-18 \end{aligned}$$

The allocation problem now can be stated in either of two ways: (1) for a given level of average network power, $p = \underline{w}^T \underline{m}$, minimize the network variance $\sigma^2 = \underline{w}^T P \underline{w}$; (2) for a given allowable network variance $\sigma_a^2 = \underline{w}^T P \underline{w}$, maximize the average network power $p = \underline{w}^T \underline{m}$. Consider case 1 first.

This is a constrained minimization problem, where it is desired to minimize

$$f(\underline{w}) = \underline{w}^T P \underline{w} \quad 2-19$$

subject to the constraint

$$p_a = \underline{w}^T \underline{m} \quad 2-20$$

This can be solved using the method of Lagrange multipliers; let

$$\begin{aligned} g(\underline{w}) &= f(\underline{w}) + \lambda(p_a - \underline{w}^T \underline{m}) \\ &= \underline{w}^T P \underline{w} + \lambda(p_a - \underline{w}^T \underline{m}) \quad 2-21 \end{aligned}$$

Then

$$\frac{\partial g}{\partial \underline{w}} = 2\underline{w}^T P - \lambda \underline{m}^T \quad 2-22$$

Now the partial derivative (eq. 2-22) must be set equal to zero, which, along with eq. 2-20, allows the minimizing value of \underline{w} to be found.

Solving 2-22 for \underline{w} yields

$$\underline{w} = \frac{\lambda}{2} P^{-1} \underline{m} \quad 2-23$$

Substitute this into eq. 2-20,

$$p_a = \frac{\lambda}{2} \underline{m}^T P^{-1} \underline{m} \quad 2-24$$

which can be solved for λ since P (and hence P^{-1}) is positive definite.

$$\lambda = \frac{2 p_a}{\underline{m}^T P^{-1} \underline{m}} \quad 2-25$$

Substitution of eq. 2-25 into 2-23 gives the required value for \underline{w} :

$$\underline{w} = p_a \frac{P^{-1} \underline{m}}{\underline{m}^T P^{-1} \underline{m}} \quad 2-26$$

This value of \underline{w} is an extremal of the function in eq. 2-19. For this to be a minimum, the second partial derivative must be positive semi-definite. Thus, we want

$$\frac{\partial^2 f}{\partial \underline{w}^2} \geq 0$$

or

$$P \geq 0 \quad 2-27$$

Since

$$P = E(\underline{x} - \underline{m})(\underline{x} - \underline{m})^T \quad 2-28$$

for any vector \underline{r} (of dimension equal to the dimension of \underline{x}), if

$$\underline{r}^T P \underline{r} \geq 0 \quad 2-29$$

then inequality 2-27 is satisfied. Applying given relationships and regrouping gives

$$\begin{aligned}
\underline{r}^T \underline{P} \underline{r} &= \underline{r}^T [E(\underline{x} - \underline{m})(\underline{x} - \underline{m})^T] \underline{r} \\
&= E[\underline{r}^T (\underline{x} - \underline{m})(\underline{x} - \underline{m})^T \underline{r}] \\
&= E(s^2), \text{ where } s = \underline{r}^T (\underline{x} - \underline{m}) \\
&\geq 0
\end{aligned}
\tag{2-30}$$

Thus, inequality 2-27 is satisfied and eq. 2-26 does give the minimum of eq. 2-19. Note that in eq. 2-26, the desired average network power production p_a simply scales the weighting vector \underline{w} but does not affect the percentage of windmills at each site. Without loss of generality it is convenient during analysis to set p_a equal to 1, and then to present the optimum weighting vector \underline{w} in a normalized form, where each element of \underline{w} is the fraction of total windmills at that site.

The minimum network variance can readily be determined using the optimum weighting vector given by eq. 2-26.

$$\begin{aligned}
\sigma^2 &= \underline{w}^T \underline{P} \underline{w} \\
&= p_a \frac{\underline{m}^T \underline{P}^{-1} \underline{P} \underline{P}^{-1} \underline{m} p_a}{(\underline{m}^T \underline{P}^{-1} \underline{m})^2} \\
&= \frac{p_a^2}{\underline{m}^T \underline{P}^{-1} \underline{m}}
\end{aligned}
\tag{2-31}$$

Now consider the second case, which is to maximize the average network power production while holding the network variance fixed. The problem is to maximize

$$p = \underline{w}^T \underline{m} \tag{2-32}$$

subject to the constraint

$$\sigma_a^2 = \underline{w}^T \underline{P} \underline{w} \tag{2-33}$$

This problem is solved in the same manner as in the previous case, using Lagrange multipliers. The augmented function is given by

$$g(\underline{w}) = \underline{w}^T \underline{m} + \lambda(\sigma_a^2 - \underline{w}^T P \underline{w}) \quad 2-34$$

Then the partial derivative of g is

$$\frac{\partial g}{\partial \underline{w}} = \underline{m}^T - 2\lambda \underline{w}^T P \quad 2-35$$

Setting this equal to zero yields

$$\underline{w} = \frac{1}{2\lambda} P^{-1} \underline{m} \quad 2-36$$

Substitute into eq. 2-23

$$\begin{aligned} \sigma_a^2 &= \frac{\underline{m}^T P^{-1} P P^{-1} \underline{m}}{4\lambda^2} \\ &= \frac{\underline{m}^T P^{-1} \underline{m}}{4\lambda^2} \end{aligned} \quad 2-37$$

or

$$\begin{aligned} 4\lambda^2 &= \frac{\underline{m}^T P^{-1} \underline{m}}{\sigma_a^2} \\ 2\lambda &= \pm \left[\frac{\underline{m}^T P^{-1} \underline{m}}{\sigma_a^2} \right]^{1/2} \end{aligned} \quad 2-38$$

To determine the correct sign to use in eq. 2-37 before substituting into eq. 2-36, consider the second partial derivative. For a maximum to occur, the following must hold:

$$\frac{\partial^2 g}{\partial \underline{w}^2} \leq 0 \quad 2-39$$

Since

$$\frac{\partial^2 g}{\partial \underline{w}^2} = -2\lambda P \quad 2-40$$

for inequality 2-30 to be satisfied, we must have

$$\lambda \geq 0 \quad 2-41$$

Applying this to eq. 2-38 gives the value for λ , which is

$$\lambda = \frac{1}{2\sigma_a} (\underline{m}^T P^{-1} \underline{m})^{1/2} \quad 2-42$$

Substitute this into eq. 2-36 to obtain

$$\underline{w} = \sigma_a \frac{P^{-1} \underline{m}}{(\underline{m}^T P^{-1} \underline{m})^{1/2}} \quad 2-43$$

The allowable standard deviation σ_a of network power fluctuations plays the same role in this case of maximizing average power as the desired average power p_a in the case of minimizing network power fluctuations. Both scale the weighting vector \underline{w} , but do not affect the relative weighting from site to site.

The maximum average network power can be determined using the weighting given in eq. 2-43.

$$\begin{aligned} p &= \underline{w}^T \underline{m} \\ &= \sigma_a \frac{\underline{m}^T P^{-1} \underline{m}}{(\underline{m}^T P^{-1} \underline{m})^{1/2}} \\ &= \sigma_a (\underline{m}^T P^{-1} \underline{m})^{1/2} \end{aligned} \quad 2-44$$

A summary of the results for determining the optimum weighting vector is presented in Table 2-1. Note that both cases give the same

relative weightings, since the weighting vector is proportional to $P^{-1}\underline{m}$. If there is no correlation in the variations from site to site, then the elements of the weighting vector \underline{w} are proportional to the ratio of the mean power to the variance at each site. A larger variance reduces the number of wind generators allocated to that site, while a greater mean power increases the number. The derived weighting vector satisfies our intuition about how wind generators should be allocated. Of course, in the realistic case of non-zero correlations, it is difficult to establish such a simple relationship; however, the general trend described above still holds.

Table 2-1. Summary of Optimal Weightings

	CASE 1	CASE 2
OBJECTIVE	Given average power p_a , minimize network variance	Given network variance σ_a^2 maximize average network power
WEIGHTING	$\underline{w} = p_a \frac{P^{-1} \underline{m}}{\underline{m}^T P^{-1} \underline{m}}$	$w = \sigma_a \frac{P^{-1} \underline{m}}{\sqrt{\underline{m}^T P^{-1} \underline{m}}}$
VARIANCE	$\sigma^2 = \frac{p_a^2}{\underline{m}^T P^{-1} \underline{m}}$	σ_a^2
AVG. POWER	p_a	$p = \sigma_a \sqrt{\underline{m}^T P^{-1} \underline{m}}$

Non-Negative Constraints

The weighting vectors shown in Table 2-1 are optimal in that they do minimize or maximize the stated functions. However, an additional factor that must be taken into account is that each element of the weighting vector \underline{w} must be non-negative in order to be physically realizable.

The problem statement for minimizing the network variance, for example, must be modified as follows to include the non-negative constraint. The corresponding problem statement for maximizing the network power could be modified similarly.

Determine the weighting vector \underline{w} to minimize the network variance

$$\sigma^2 = \underline{w}^T P \underline{w} \quad 2-45$$

subject to the constraints that the specified average network power p_a is given by

$$p_a = \underline{w}^T \underline{m} \quad 2-46$$

and that the elements of the weighting vector \underline{w} be non-negative:

$$w_i \geq 0, \quad i = 1, 2, \dots, n \quad 2-47$$

The non-negative constraint introduces sufficient complication that an analytical solution to the minimization problem cannot be obtained in general. Mathematical programming techniques (24,25,26) can be applied however, to obtain a numerical solution using numerical values for covariance matrix P , mean value vector \underline{m} , and the average power level p_a .

The augmented function with Lagrange multiplier λ is

$$g(\underline{w}) = \underline{w}^T P \underline{w} + \lambda(\underline{w}^T \underline{m} - p_a)$$

and define \underline{v} to be the gradient of g

$$\underline{v} = \frac{\partial g}{\partial \underline{w}} = 2P\underline{w} + \lambda \underline{m}$$

Now the set of linear equations that must be solved is

$$\underline{w}^T \underline{m} = p_a$$

$$2P\underline{w} - \underline{v} + \lambda \underline{m} = 0$$

$$\underline{w}^T \underline{v} = 0$$

with the conditions

$$\underline{w} \geq 0, \underline{v} \geq 0$$

These equations can be solved for the unknown variables \underline{w} , \underline{v} and λ using linear programming techniques, with the condition that only one term of the set w_i and v_i , for each i , can be non-zero at any given iteration.

The results presented in this study include the application of mathematical programming (in this case, quadratic programming) methods to insure non-negative weightings. Weightings were first determined without constraining the weights to be non-negative, then if some weights were negative, the minimization problem was resolved using the numerical techniques.

Examples of Optimal Weightings

As an example of the potential benefit of using an optimal method of allocating numbers of wind generators to various sites, an arbitrary three-site system will be analyzed. Primarily, we will be interested in the amount by which the network variance can be reduced from that obtained by equal allocation among the sites. These variances can in turn

be compared to the network variance that would result if all the wind generators were clustered at a single site.

The three-site system was chosen to have a covariance matrix of the form:

$$P = \begin{bmatrix} \sigma_1^2 & c_{12} \sigma_1 \sigma_2 & 0 \\ c_{12} \sigma_1 \sigma_2 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} \quad 2-48$$

where c_{12} is the correlation coefficient ($-1 \leq c_{12} \leq 1$) between the wind power sequences for sites one and two. For selected sets of the variances σ_1^2 , σ_2^2 and σ_3^2 and the mean values chosen for each site, the correlation coefficient c_{12} was varied in increments. The optimal weightings and equal weightings were then computed and used to determine the network variances, while the average power production was held at the same constant value. The appropriate equations are summarized below:

$$p_a = \text{average power} = \underline{w}^T \underline{m} \quad 2-49$$

optimal case

$$\text{weighting} = \underline{w}_{\text{opt}} = p_a \frac{P^{-1} \underline{m}}{\underline{m}^T P^{-1} \underline{m}} \quad 2-50$$

$$\text{variance} = \sigma_{\text{opt}}^2 = \underline{w}_{\text{opt}}^T P \underline{w}_{\text{opt}}$$

$$= \frac{p_a^2}{\underline{m}^T P^{-1} \underline{m}} \quad 2-51$$

equal case

$$\text{weighting} = \underline{w}_{\text{eq}} = \frac{p_a}{\sum_i m_i} \underline{u}, \text{ where } \underline{u}^T = [1 \ 1 \dots 1] \quad 2-52$$

$$\text{variance} = \sigma_{\text{eq}}^2 = \underline{w}_{\text{eq}}^T P \underline{w}_{\text{eq}} \quad 2-53$$

Since the average power p_a scales both cases in an identical manner, and the comparison between the two cases is of interest rather than absolute values, p_a was set equal to one in the following evaluation. Four situations were examined: (a) mean values and variances the same from site to site; (b) mean values different but variances the same; (c) mean values the same but variances different; and (d) both means and variances different from site to site.

Configuration (a), means and variances the same for all sites, showed very little reduction, in general, in the network variance by using optimal weightings compared to equal weightings. The only significant improvement was achieved for a large negative correlation, with correlation coefficient c_{12} approximately in the range -1 to -0.5. This is obviously due to direct cancelling of fluctuations when power from two highly negatively correlated sites is combined. Of course, this is a desirable result; however, it is not a likely event. In the range of the correlation coefficient

$$|c_{12}| \leq 0.3$$

there is less than a 3% improvement in the network variance by using optimal weightings instead of equal weightings. This percentage improvement is unaffected by changes in the levels of the means and

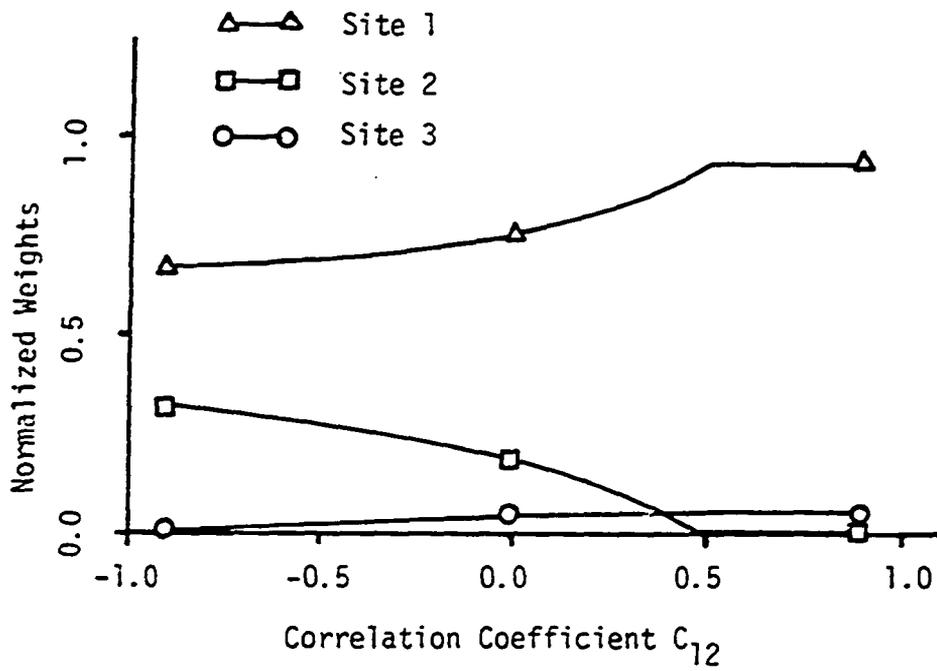
variances, as long as all the means remain the same from site to site and the variances do also. The more interesting cases dealing with different means and variances from site to site are examined in the following. It is shown that there can be a significant reduction in the network power fluctuations if optimal, and not equal, weightings are used.

Table 2-2 shows the various cases for which results are presented. For each case, the network variance was calculated as a function of the correlation coefficient c_{12} (see eq. 2-48).

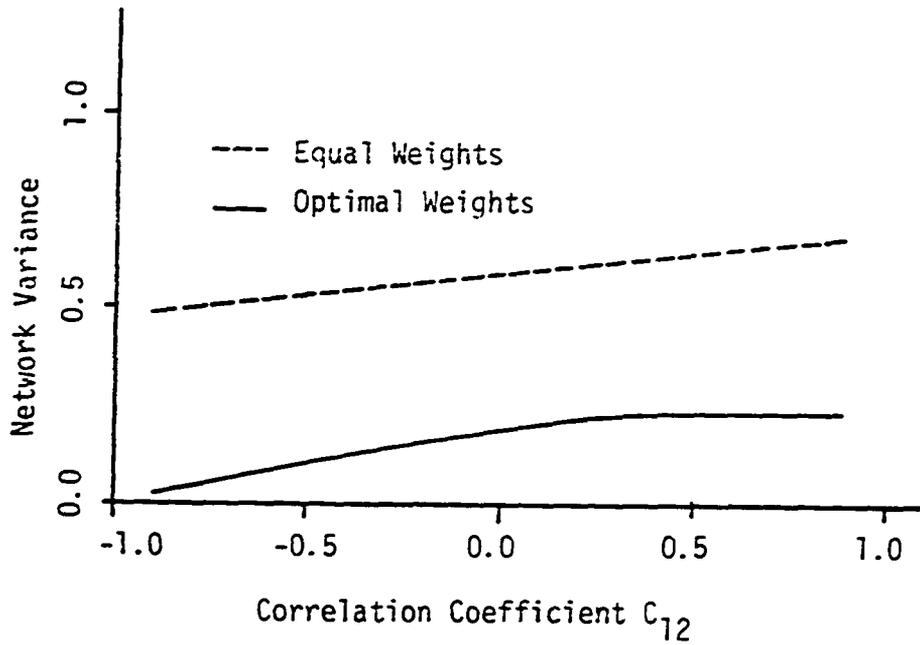
Table 2-2. Means and Standard Deviations for Examples

Case No.	Site Means	Standard Deviations
1	10, 10, 10	5, 10, 20
2	10, 10, 10	20, 10, 5
3	5, 10, 20	10, 10, 10
4	20, 10, 5	10, 10, 10,
5	5, 10, 20	5, 10, 20
6	5, 10, 20	20, 10, 5
7	20, 10, 5	5, 10, 20
8	20, 10, 5	20, 10, 5

Figures 2-1 and 2-2 show the network variances for Cases 1 and 2. The substantial reduction in the network variance by using optimal weightings is primarily due to the optimal weightings being roughly proportional to the inverse of the site variances, respectively. The additional reduction of the network variance that is achieved by accounting for the correlation (shown by correlation coefficient c_{12}) is rather small. The network variance for both sets of weights decreases with

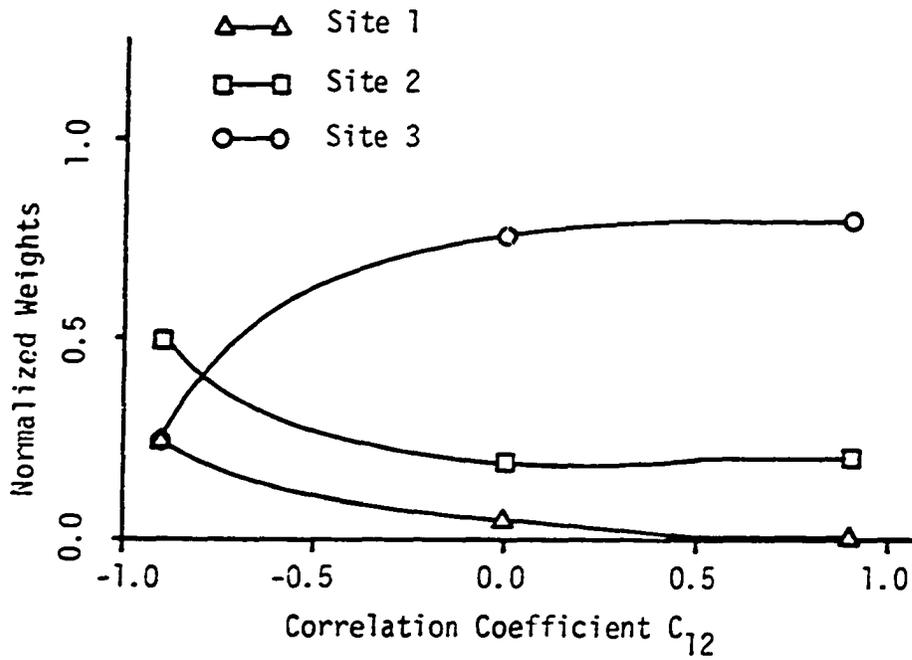


a) Optimal Weights

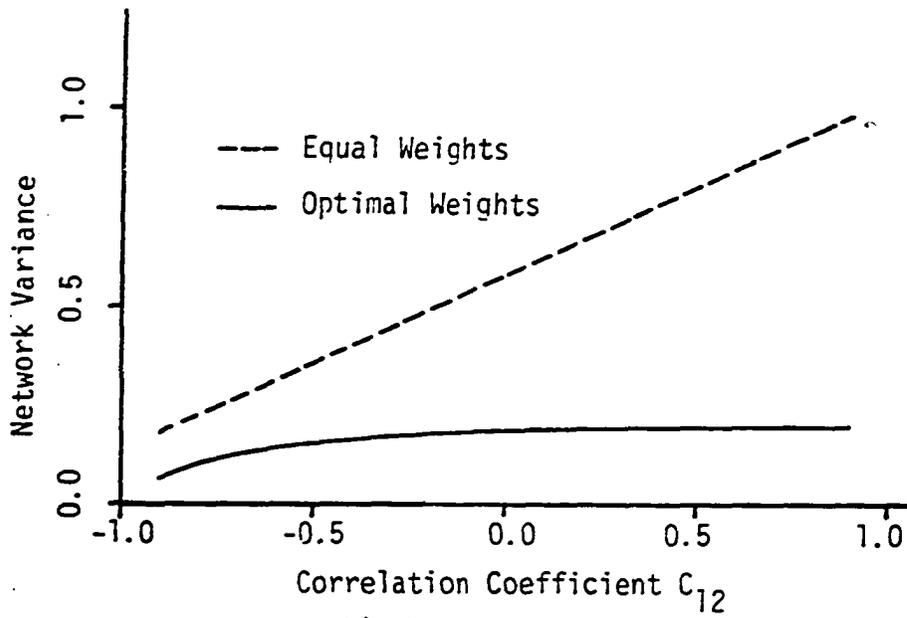


b) Network Variance

Fig. 2-1. Results for Example Case 1



a) Optimal Weights



b) Network Variance

Fig. 2-2. Network Variance for Example Case 2

decreasing c_{12} , but the sensitivity to changes in c_{12} is different for the two cases. Figure 2-2 shows that there is little effect on the optimal network variance for changes in c_{12} , except for large negative values. As c_{12} increases positively, sites 1 and 2 tend to become the equivalent of a single site, with a variance much larger than that for site 3. The optimal weighting is to simply depend primarily on site 3 and to disregard (i.e., small weight) sites 1 and 2.

Figure 2-1 shows that the optimal network variance is more sensitive to changes in c_{12} , but is nevertheless substantially smaller than the network variance using equal weighting. In the vicinity of c_{12} equal to zero, approximately 67% reduction in the network variance can be achieved by using optimal instead of equal weighting. Also note that by simply combining the power from several sites, whether done optimally or not, the variance can be significantly reduced from that for an individual site producing the same average power. For example, for Cases 1 and 2 the means are all equal to ten, so that to produce a power equal to one from only one site, the weighting would be 0.1. Combining this with the standard deviations of 5, 10, and 20 produces variances from each site individually of 0.25, 1.0, and 4.0, respectively. For the second and third variances, the individual variances are significantly more than the network variance with either equal or optimal weighting, which are not more than about 0.7 and 0.3, respectively (see Fig. 2-1).

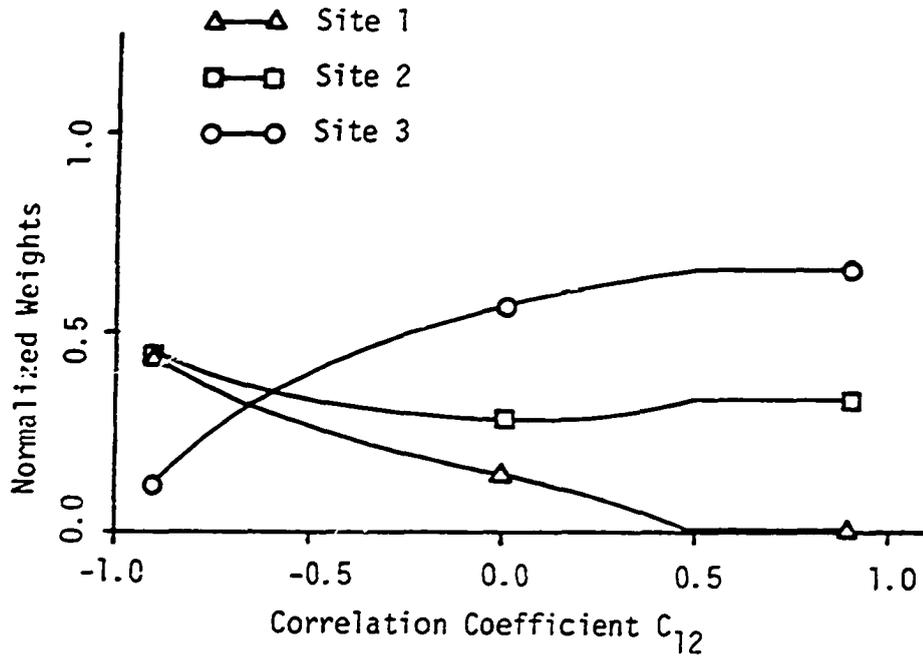
When the means are allowed to vary from site to site but the variances are held constant, the improvement obtained by using optimal rather than equal weightings is not quite so dramatic as with

the previous cases. However, significant reduction in the network variance can still be achieved. Figures 2-3 and 2-4 show the network variance for the optimal and equal weightings for Cases 3 and 4. Figure 2-4 shows that significant improvement is obtained over all the range of c_{12} ; there is a 25% reduction in network variance at c_{12} equal to 0.4, and as c_{12} decreases, the percentage improvement increases. When the means from site to site are varied in increasing order (Case 3; Fig. 2-3) rather than decreasing order (Case 4; Fig. 2-4) there is significant improvement for positive values of c_{12} . However, as c_{12} goes negative, there is less difference between the network variances for the optimal and equal weightings, and for c_{12} in the range of -0.7 to -0.3 there is less than a 3% improvement in network variance.

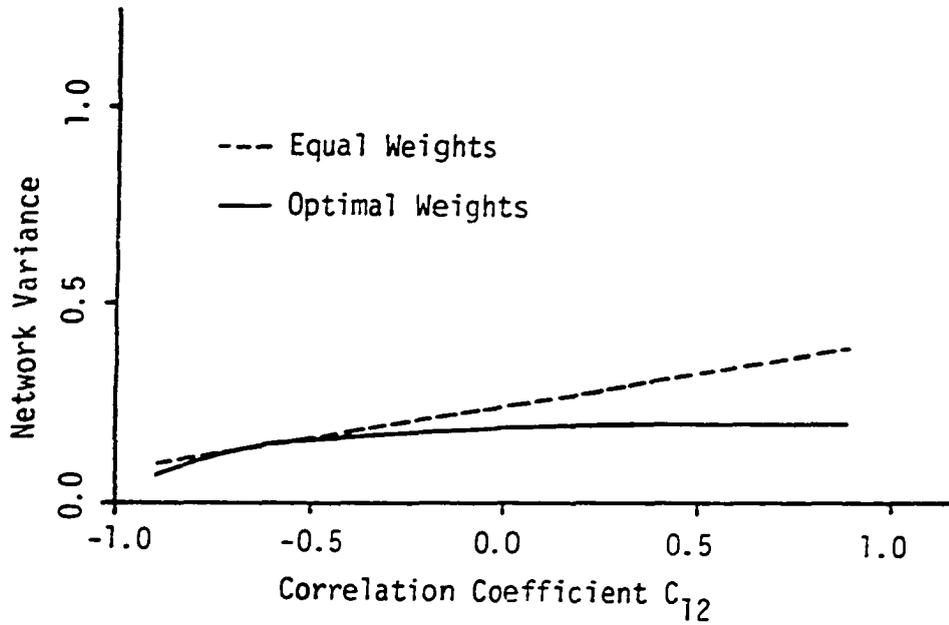
When both the means and variances are varied from site to site, there can be substantial reduction in the network variance by using optimal rather than equal weightings. Figures 2-5 through 2-8 show the results for such sets of values, which correspond to Cases 5 through 8, respectively. In all of these cases, a significant reduction in network variance is achieved whenever c_{12} is in the vicinity of zero. Very large variance reductions occur for Cases 6 and 7, shown in Figs. 2-6 and 2-7. For the other two (Cases 5 and 8) somewhat less but still significant, improvement is obtained. The variance reduction for c_{12} equal to zero ranges from 22% for Cases 5 and 8 to 86% for Cases 6 and 7.

Arbitrary Demand Profile

In the previous section, examples are presented demonstrating the benefit of using optimal site weightings, which resulted in minimizing

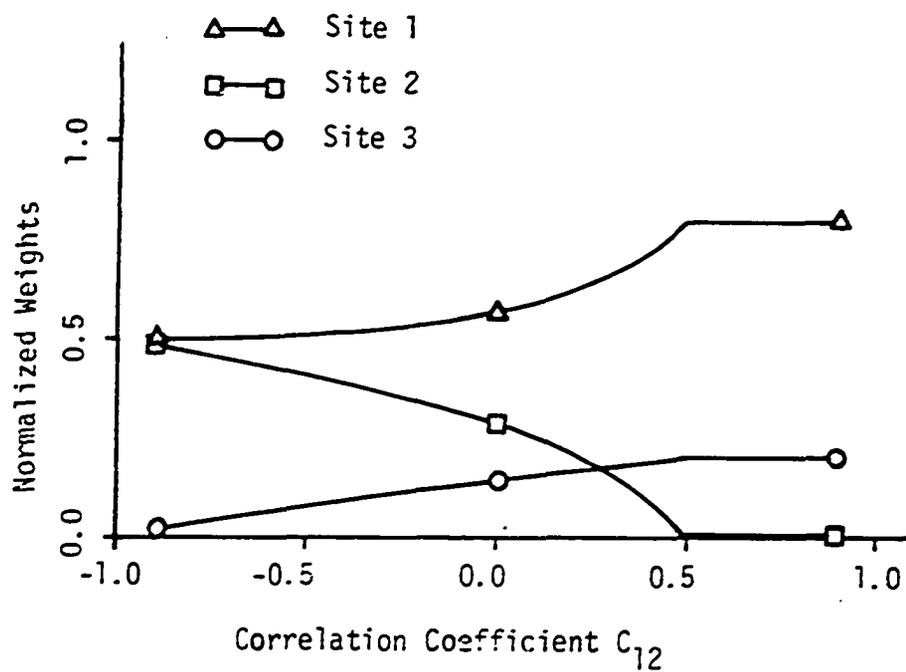


a) Optimal Weights

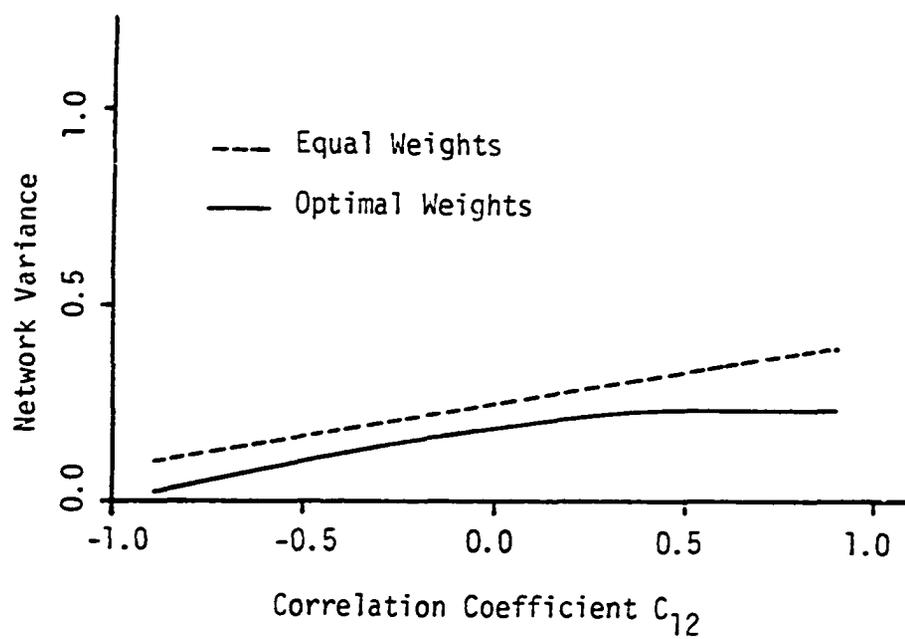


b) Network Variance

Fig. 2-3. Network Variance for Example Case 3

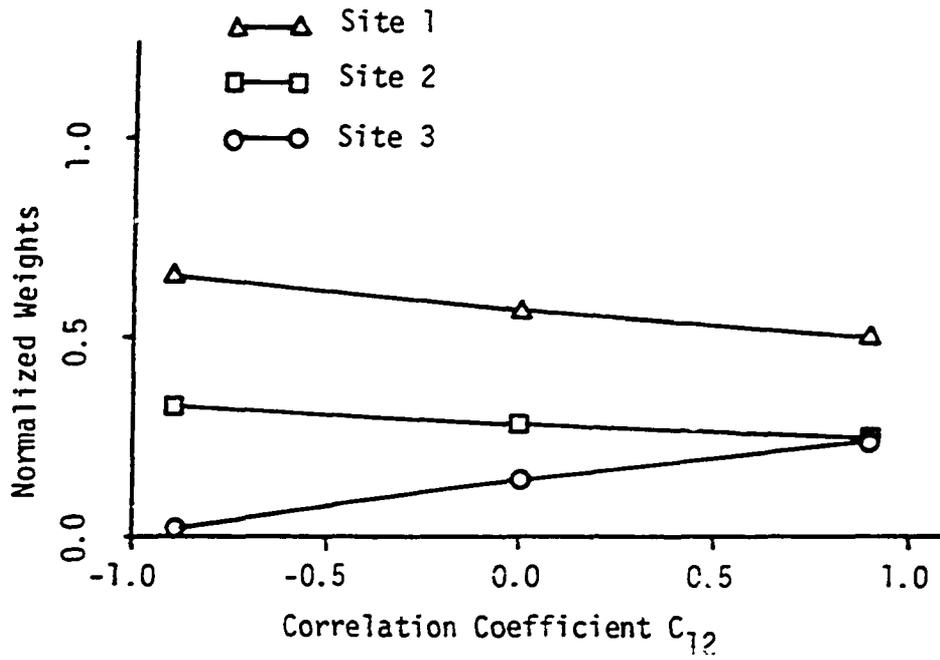


a) Optimal Weights

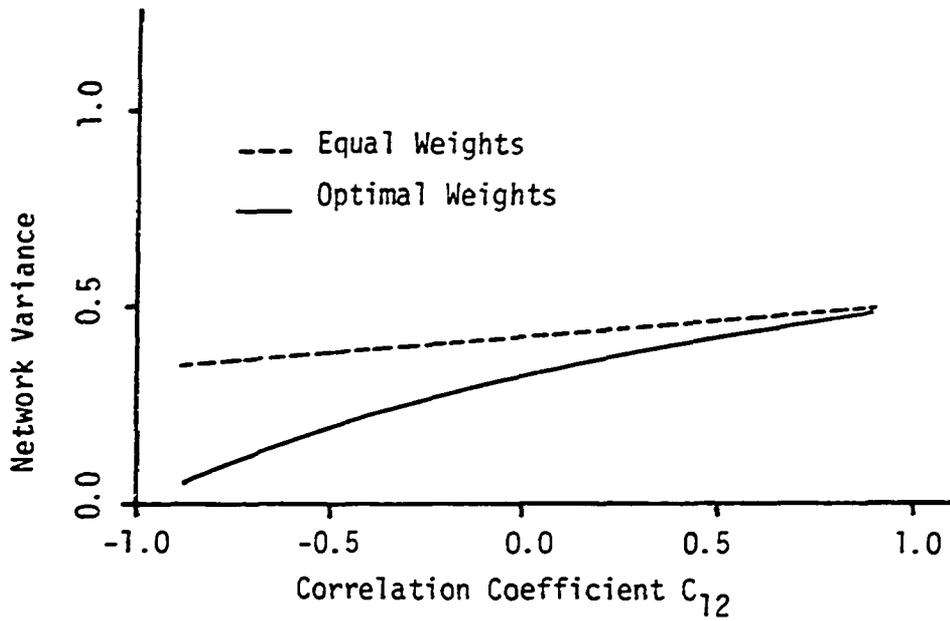


b) Network Variance

Fig. 2-4. Network Variance for Example Case 4



a) Optimal Weights



b) Network Variance

Fig. 2-5. Network Variance for Example Case 5

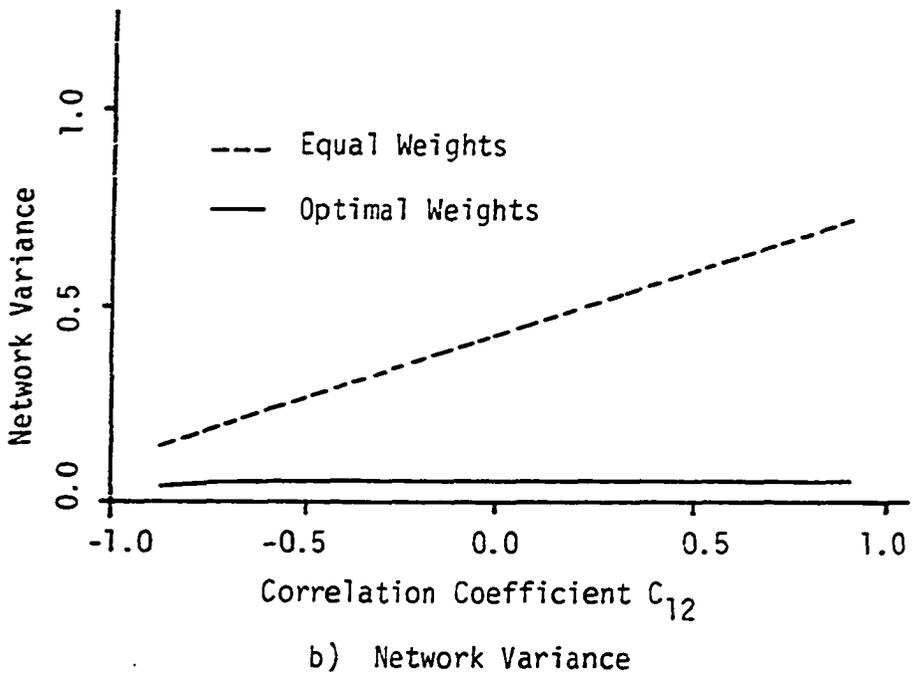
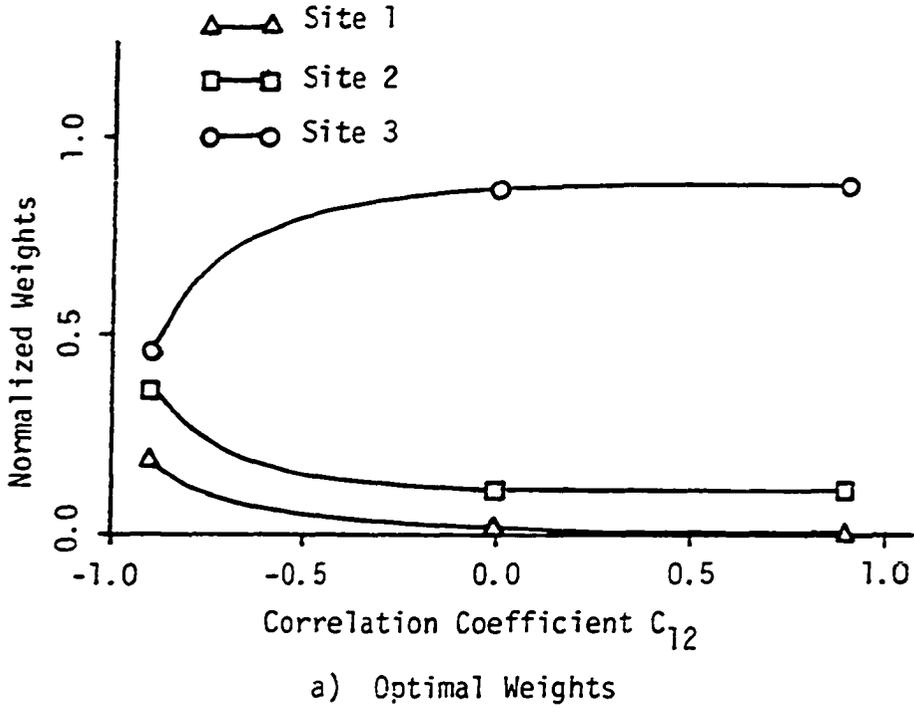
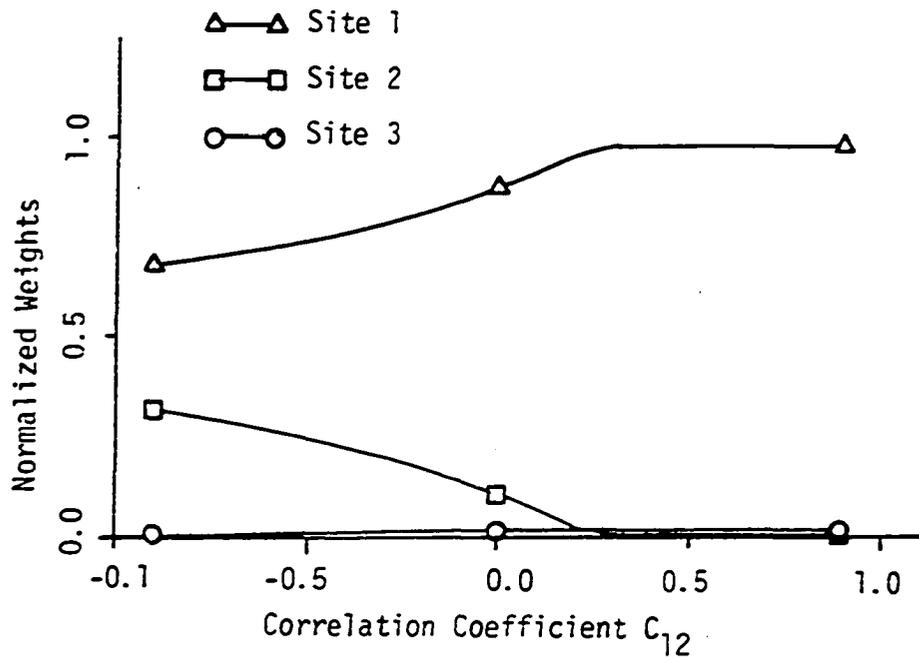
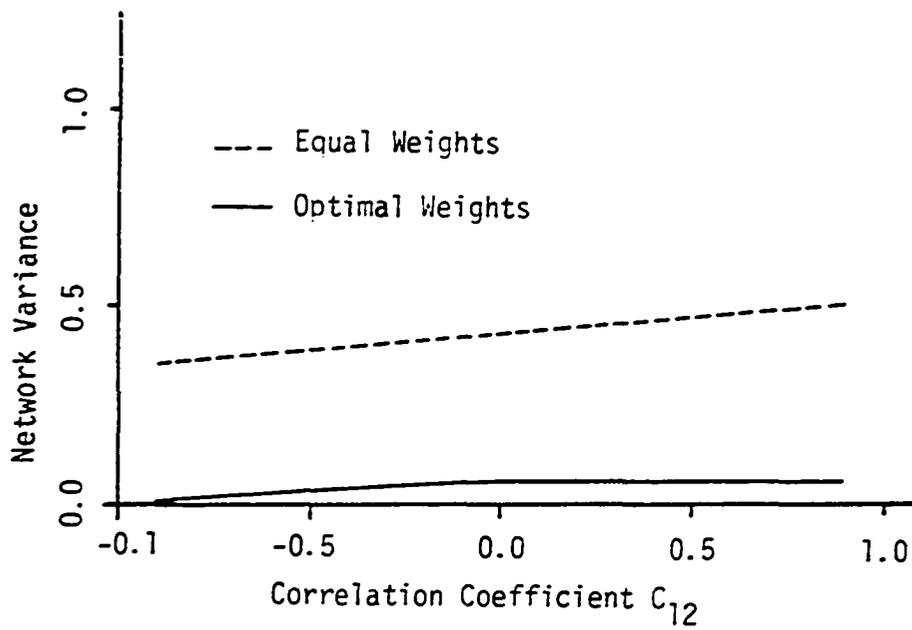


Fig. 2-6. Network Variance for Example Case 6

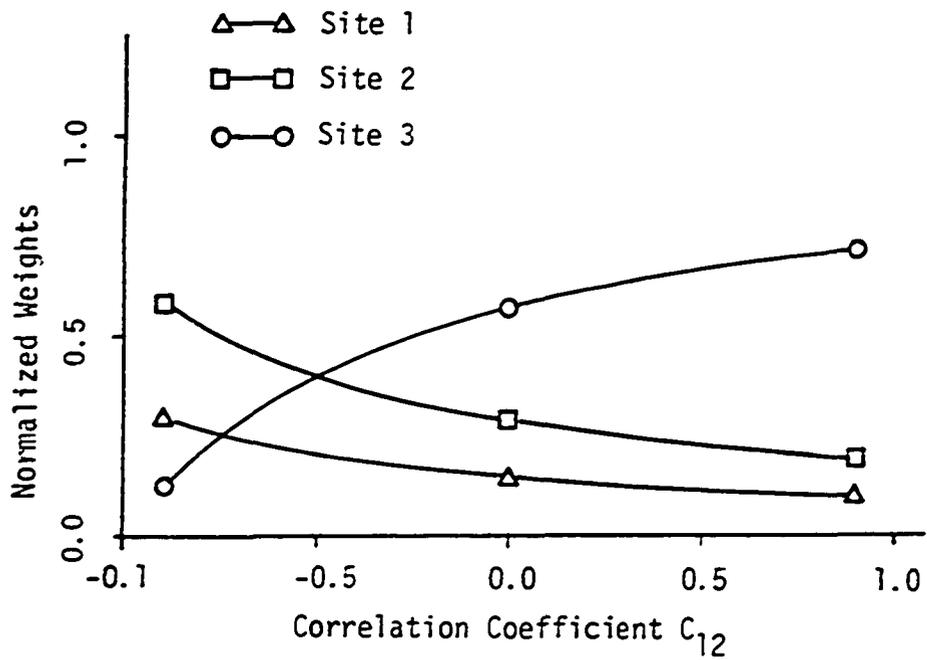


a) Optimal Weights

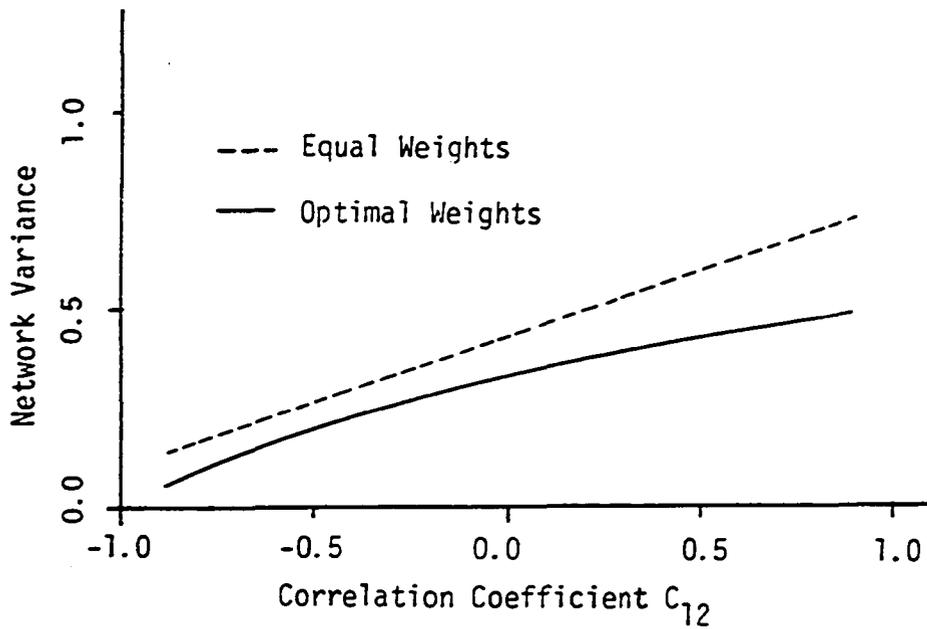


b) Network Variance

Fig. 2-7. Network Variance for Example Case 7



a) Optimal Weights



b) Network Variance

Fig. 2-8. Network Variance for Example Case 8

the network variance. That variance is a measure of the fluctuations about the mean power level; the network performance is thus being driven toward a constant power production level. The wind-generated power may be more useful if the power level followed some arbitrary (but perhaps not constant) demand profile. For example, a typical 24-hour load history for an electric utility may be as shown in Fig. 2-9, with the desired wind generated power shown as the dashed line. In this case, it is desirable to implement the wind generator system so as to minimize

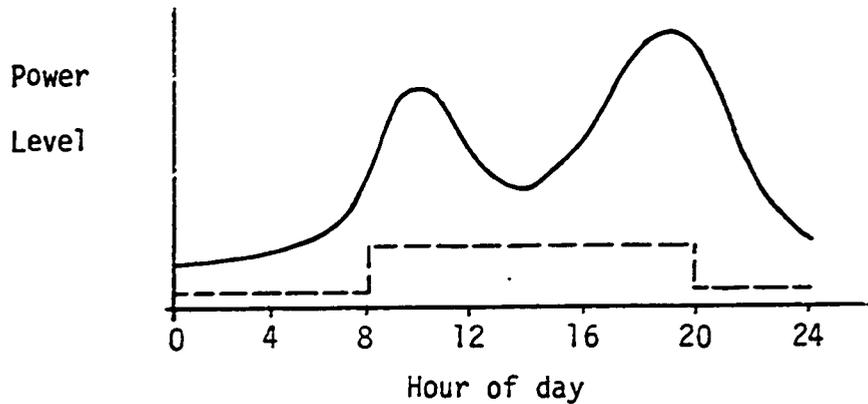


Fig. 2-9. Typical Power Demand Profile

the fluctuations about the dashed line. This may be accomplished as follows.

Let the wind generator power level between 8 and 20 hours be p_1 and let p_2 be the power level for the remaining hours. Then for the site weighting vector given by \underline{w} , we must have

$$p_1 = \underline{w}^T \underline{m}_1$$

and

$$p_2 = \underline{w}^T \underline{m}_2$$

2-54

where \underline{m}_i , $i = 1, 2$, is the mean wind power during the two time intervals. If r_1 and r_2 are the deviations of the network power about the desired level during the respective time periods, then

$$\begin{aligned} r_1 &= \underline{w}^T \underline{x}_1 - \underline{w}^T \underline{m}_1 \\ r_2 &= \underline{w}^T \underline{x}_2 - \underline{w}^T \underline{m}_2 \end{aligned} \tag{2-55}$$

where \underline{x} is the wind power at the various sites. The variances of r_1 and r_2 can now be determined and combined to produce the average variance v .

$$\begin{aligned} v &= \frac{20 - 8}{24} E(r_1^2) + \frac{8 + 24 - 20}{24} E(r_2^2) \\ &= \frac{1}{2} \underline{w}^T P_1 \underline{w} + \frac{1}{2} \underline{w}^T P_2 \underline{w} \\ &= \underline{w}^T \left(\frac{1}{2} P_1 + \frac{1}{2} P_2 \right) \underline{w} \end{aligned} \tag{2-56}$$

where P_1 and P_2 are the covariance matrices for the two time periods. Now the problem is to find the weightings \underline{w} that minimize eq. 2-56, subject to the constraints in eq. 2-54. Furthermore, it may be necessary to apply the non-negative constraint

$$w_i \geq 0, \quad i = 1, 2, \dots, n \tag{2-57}$$

to assure that the impossible case of negative weightings is avoided. Ignoring this last constraint for the moment, the minimization problem may be solved in a manner similar to that presented earlier for computing optimal weightings, except that now two constraints (eq. 2-54) must be included.

Using the Lagrange multiplier method, the augmented function to be minimized is

$$g(\underline{w}) = \underline{w}^T P \underline{w} + \lambda_1 (P_1 - \underline{w}^T \underline{m}_1) + \lambda_2 (P_2 - \underline{w}^T \underline{m}_2) \quad 2-58$$

where

$$P = \frac{1}{2} P_1 + \frac{1}{2} P_2$$

and λ_1 and λ_2 are the Lagrange multipliers. Differentiating eq. 2-58 with respect to \underline{w} , setting equal to zero, and solving for \underline{w} gives

$$\underline{w} = \frac{1}{2} P^{-1} (\lambda_1 \underline{m}_1 + \lambda_2 \underline{m}_2) \quad 2-59$$

This is substituted into eq. 2-54, and the resulting expressions are solved for λ_1 and λ_2 , which are substituted into eq. 2-59 to obtain the optimal value of \underline{w} .

$$\underline{w} = \frac{P^{-1} (\underline{m}_2^T P^{-1} \underline{m}_2 P_1 - \underline{m}_1^T P^{-1} \underline{m}_2 P_2) \underline{m}_1 + P^{-1} (-\underline{m}_2^T P^{-1} \underline{m}_1 P_1 + \underline{m}_1^T P^{-1} \underline{m}_1 P_2) \underline{m}_2}{(\underline{m}_1^T P^{-1} \underline{m}_1) (\underline{m}_2^T P^{-1} \underline{m}_2) - (\underline{m}_1^T P^{-1} \underline{m}_2)^2} \quad 2-60$$

If any of the individual site weightings w_i given by eq. 2-60 are negative, then the constraint in expression 2-57 must be applied using mathematical programming techniques. This requires iterative numerical processes and prevents showing the optimal weightings in a closed form expression, as in eq. 2-60. Nevertheless, the optimal weightings can still be obtained.

In the above example, only two power levels were considered. The same approach for the solution can be readily expanded to many levels, thereby allowing an arbitrary demand profile to be used. It must be recognized, however, that no scheme can eliminate fluctuations in power from a wind generator network; the fluctuations can only be reduced somewhat by optimal allocation methods. This suggests that highly

detailed power demand profiles should be avoided for a wind network.

Seasonal changes in the demand profile can also be incorporated, with the objective being to choose the weightings to minimize the average deviations for the year about a desired pattern. If it is more important to meet a particular profile during one season than another, the fluctuations during that critical season can be given greater weight than for other seasons, prior to the minimization.

Generating Capacity

Due to the uncertainties in the wind power levels, wind generators tend to be disregarded as sources of additional generating capacity. Minimizing the fluctuations in the power from a wind generator network, by optimal site selection and establishing a network over an area of differing wind regimes, tends to improve the generation capacity of the wind network. The variance of the wind power gives an overall measure of average fluctuations, but to examine the improvement in generation capacity, additional factors must be considered. Primarily, this requires an evaluation of the duration of various levels of wind power, with particular emphasis on the duration of low power levels. Extremely high wind speeds are undesirable also, due to causing shut-downs, but it is the time duration of low wind speeds that causes the greatest effect on loss of generating capacity.

Wind power duration curves can be computed for individual sites and for various combinations of sites. From these the fraction of time can be determined for which the network power remains above a specified level. If the network is expanded to include additional wind generator

sites, there should be an increase in the time that a minimum power level is maintained. This effect can serve as a criterion in selecting additional sites to be included in a wind generator network. This aspect is explored in Chapter 4.

Reduced Number of Sites

In addition to the optimal allocation of wind generators throughout a network of sites, which is the method developed earlier in this chapter, the performance of a reduced number of sites will be examined in the following. It is likely that wind speed data are available from more sites than are to be used for wind generator installations. The problem then is to select the "best" n sites out of a possible N sites, where n is less than N , and then to allocate the wind generators optimally among the n -site network.

Let the wind power from the N sites be \underline{x} , and let \underline{w} be some weighting (i.e., wind generator allocation) for the n sites. If P is the covariance matrix of \underline{x} ,

$$\begin{aligned} P &= \text{cov}(\underline{x}) \\ &= E(\underline{x} - \underline{m})(\underline{x} - \underline{m})^T \end{aligned} \quad 2-61$$

then the wind power variance from the n -site network is given by

$$v = \underline{w}^T T^T P T \underline{w} \quad 2-62$$

where T is an appropriate matrix of ones and zeros. For example, if the reduced network consists of the first n out of N sites, then the T matrix can be written in partitioned form as

$$T = \begin{bmatrix} I \\ \text{---} \\ 0 \end{bmatrix} \quad 2-63$$

where I is an n by n identity matrix and the overall dimension of T is N by n . Writing P in partitioned form also as

$$P = \begin{bmatrix} P_{11} & \vdots & P_{12} \\ \text{---} & \text{---} & \text{---} \\ P_{21} & \vdots & P_{22} \end{bmatrix} \quad 2-64$$

results in the n -site network variance v being given by

$$v = \underline{w}^T P_{11} \underline{w} \quad 2-65$$

The method presented earlier can be used directly to find the \underline{w} that minimizes eq. 2-65.

An examination of the performance of a reduced number of sites may reveal only a minor performance degradation from that for all sites included. This could occur if the excluded sites are strongly correlated with the remaining ones, for example. In such a case, it may be appropriate to develop only the reduced number of sites for the wind generator network. If some of the partially redundant (i.e., remaining) sites were developed, however, it could provide needed operational flexibility to allow for varying electrical demand and maintenance procedures.

Operational Considerations

A number of techniques can be employed in an operational wind generator network to minimize the network fluctuations and coordinate the power generated with other (conventional) generators. A wind generator network requires somewhat different operational methods since the energy source, the wind, cannot be directly controlled. In conventional electrical generation systems (thermal, nuclear, hydroelectric) the energy

source is readily controlled as needed. Of course, various constraints must be satisfied, such as limited fuel supplies and water flow restrictions, but the general philosophy is to adjust the energy source to satisfy the demand.

In the case of a wind generator network, however, the wind is an independent (and not a controlled) variable. Thus, the general operational approach is fundamentally different than for conventional generation. Wind generator sites can be chosen to minimize, on the average, the network power fluctuations. This has been examined earlier. After sites have been developed, various operational techniques can be employed, such as the following:

- Modulate power at individual sites for benefit of the network
- Seasonally modify generation schedule
- Use wind power forecasting techniques

Similar to the operation of conventional generation, a few wind generators at an individual site could be used to follow swings in the difference between desired and actual generation from that site. By coordinating the swing generators at multiple sites, the network power fluctuations could be moderated. This would not increase the reliability of the wind-generated power, but would tend to reduce large power swings that would otherwise adversely affect the conventional portion of the total power network.

Wind characteristics vary seasonally and should be incorporated in setting generation schedules. Seasonal effects are commonly incorporated in the scheduling of hydroelectric generation; similar procedures

could be included in wind power schedules. Typical of seasonal wind power characteristics are differences in diurnal effects (coast sites have shown strong summer time diurnal effects, with only a minor effect in winter), the magnitude of fluctuations, and means. By accommodating the naturally-occurring seasonal effects, a greater amount of wind energy will be used with the subsequent savings in conventional energy sources.

Wind power forecasting can be used to predict the amount of power that will be generated from the wind network. Short-term forecasting, in the range of a few hours up to perhaps 24 hours, is the most accurate and could be used to schedule hour-to-hour changes in plant power generation. By continually monitoring wind speeds at the various sites and updating the forecasting model, the wind power forecasts would be as accurate as possible. The forecasts also will be more accurate for the network power than for power from individual sites. This is due to the reduction in the variance of the network power as more sites are included. A forecasting model could produce updated forecasts each hour, for example, from one hour ahead up to a desired time interval. This would yield improved accuracy, on the average, as the forecasting time interval becomes smaller. For example, suppose that at 10:00 AM a forecast is made of the network wind power that will exist at 6:00 PM that evening. Then by observing the network wind speeds from 10:00 AM to 11:00 AM, an improved or updated forecast of the 6:00 PM power is made at 11:00 AM. This procedure of periodically updating the 6:00 PM forecast continues as wind speeds are observed throughout the day.

Such a technique would allow coordinated scheduling of conventional generation facilities, even though it would not entirely eliminate the uncertainties in wind-generated power.

It may be possible to use forecasting techniques over time intervals greater than 24 hours. This will depend on wind speed characteristics, local and regional weather patterns, and seasonal influences. Actually, the diurnal effect mentioned earlier that was strong in summer and weak in winter, is, essentially a long-term forecast. The problem, of course, is in the accuracy of the prediction. Knowing that a strong diurnal effect exists during a particular season is a great deal different from accurately predicting wind power levels during that season. Diurnal effects and other known wind characteristics may be incorporated in the forecasting model to improve the prediction accuracy, but not necessarily to extend the forecasting interval.

CHAPTER 3.

WIND DATA

Actual wind speed data from six sites in the Pacific Northwest were obtained for the wind generator network analysis presented in Chapter 4. The wind speed data are hourly average wind speeds for a year from each of the six sites, starting in June, 1976, and extending through May 1977. This chapter presents the means, variances, and durations of these wind speeds.

The sites selected for analysis are typical of good wind power sites in the Northwest, but are not necessarily the best possible sites. No investigation was conducted to evaluate the geographical features of the sites, such as suitable terrain for wind generator installation, extent of access roads needed, land ownership, and proximity of electrical transmission facilities. For developing satisfactory statistics from wind data, it may be necessary to evaluate more than one year of data. Other analysis ⁽²²⁾ has shown that the mean wind speeds for the year for which data are presented are somewhat less than the long term mean speeds. Thus, the wind power results shown in Chapter 4 may be a little on the conservative side.

Wind Data Sites

Figure 3-1 shows the locations of the six wind data sites. The Cape Blanco site is located directly on the Oregon coast, the sites at Augspurgen Mountain, KCIV radio tower, and Goodnoe Hills are in the vicinity of the Columbia River Gorge, which is known for its consistent wind; the Kennewick site is upriver from the Columbia Gorge but is still

1. KCIV tower
2. Goodnoe Hills
3. Wells
4. Kennewick
5. Cape Blanco
6. Augspurgen Mtn.

LOCATIONS OF SIX WIND DATA SITES

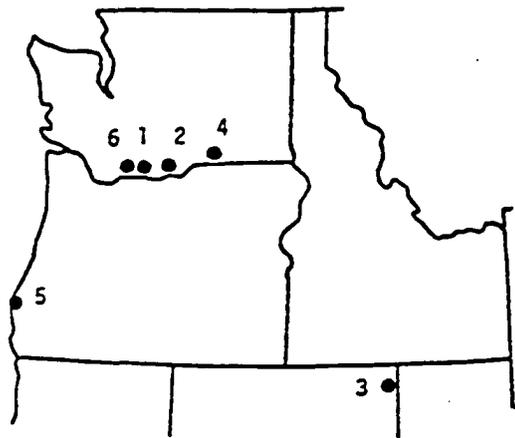


Fig. 3-1. Map of Site Locations

located near the Columbia River; the Wells, Nevada, site is in a high plateau region. The site numbers shown in Fig. 3-1 are used in the later analysis to designate the various sites.

In the Pacific Northwest, as in many regions of the United States, there is a definite lack of wind data adequate for wind power analysis. The wind speed data taken at airports and by public and private agencies for agricultural purposes, as well as wind data from other weather stations, are generally not intended to be the highest wind speeds in the area. Wind speed measurements at airports, for example, are intended for supplying information to aircraft about wind speeds in the immediate vicinity of the runway. Winds on nearby hilltops may be much greater. Since wind speeds are highly dependent on geographical features, it is crucial to locate measuring instruments at the most windy sites. Extrapolating wind speed data from the instrumented site to other non-instrumented sites is subject to large errors, especially in regions of mountainous terrain or other diverse geographical features. It is likely

that numerous high-quality wind power sites exist in the Pacific Northwest, due to the major rivers and valleys cutting swaths through which large air masses flow, but remain to be identified.

Wind Power

The power in the wind is determined by evaluating the kinetic energy of the wind, with an accounting for the mass flow rate. The result is an expression for the wind power density, which is given by

$$\frac{1}{2} \rho v^3 \quad 3-1$$

where ρ is air density and v is the wind speed. Propeller-type horizontal-axis wind generators can extract a theoretical maximum of 16/27 of this power (27,28). In addition, there are mechanical and electrical efficiencies which further reduce the usable electrical power from a wind generator. Letting e_m and e_e be those efficiencies, respectively, the electrical power produced by a wind generator is the following:

$$\text{power output} = e_a e_m e_e A \frac{1}{2} \rho v^3 \quad 3-2$$

where A is the area swept by the wind generator blades and e_a is the rotor aerodynamic efficiency, which is bounded by the value 16/27.

Typical values of the efficiencies are the following (10).

$$e_a = 0.46 \quad 3-3$$

$$e_m = 0.96 \quad 3-4$$

$$e_e = 0.95 \quad 3-5$$

Using these values, the wind generator power output is

$$\text{power output} = 0.21 A \rho v^3 \quad 3-6$$

Converting to standard units for the swept area A and the wind speed v and using a value of 1.29 kg/m^3 for the density of air under standard conditions, the power output in kilowatts is given by

$$\text{power output in kw} = 2.2 \cdot 10^{-6} A_{\text{ft}^2} v_{\text{mph}}^3 \quad 3-7$$

$$= 2.7 \cdot 10^{-4} A_{\text{m}^2} v_{\text{m/s}}^3 \quad 3-8$$

where A_{ft^2} and A_{m^2} are the swept areas in square feet and square meters, respectively, and v_{mph} and $v_{\text{m/s}}$ represent the wind speed in miles per hour and meters per second, respectively.

The annual duration curves of wind power density can be calculated from the wind speed duration curves shown in Figs. 3-2 through 3-7 for all of the six sites shown in Fig. 3-1. Substantial wind power is present at all of the sites; however, the duration curves show that there are also significant time periods of little or no wind power.

Wind turbine generators normally operate within certain limits of maximum and minimum wind speeds. A minimum wind speed is necessary to overcome friction and other losses, and above some maximum wind speed the blades are feathered so as to avoid structural damage. Also, at an intermediate wind speed, the rated power output of the electrical generator is achieved. Thus, at wind speeds above the rated wind speed, but below the shut-down speed, the electrical power produced is constant. At speeds less than rated, but above the cut-in speed, the electrical power is proportional to the cube of the wind speed as shown in eqs. 3-7 or 3-8. For this study, typical values are used which are 8 mph cut-in speed, 25 mph rated speed, and 40 mph shut-down speed. The per-unit wind generator power output for this set of values is shown in

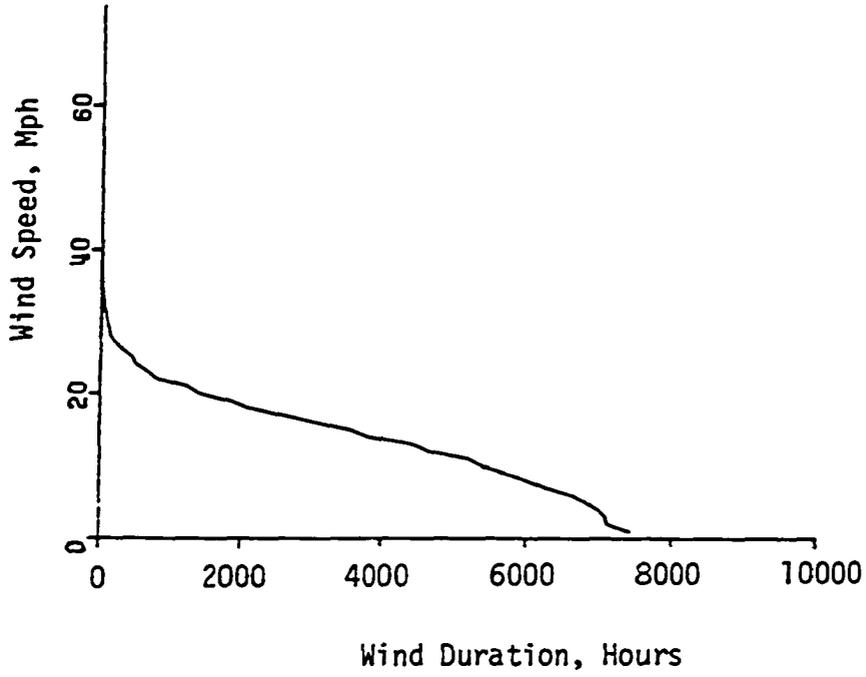


Fig. 3-2. Wind Duration for Station 1

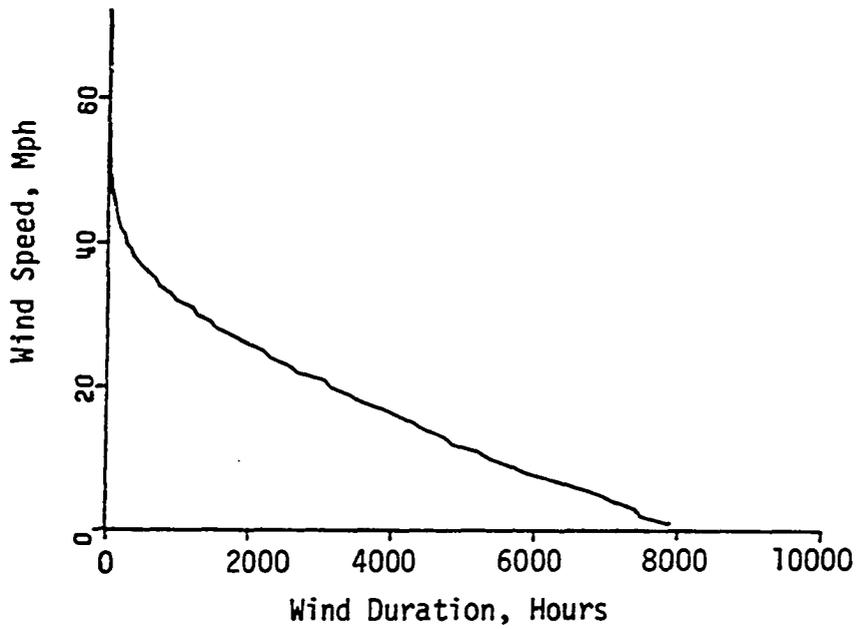


Fig. 3-3. Wind Duration for Station 2

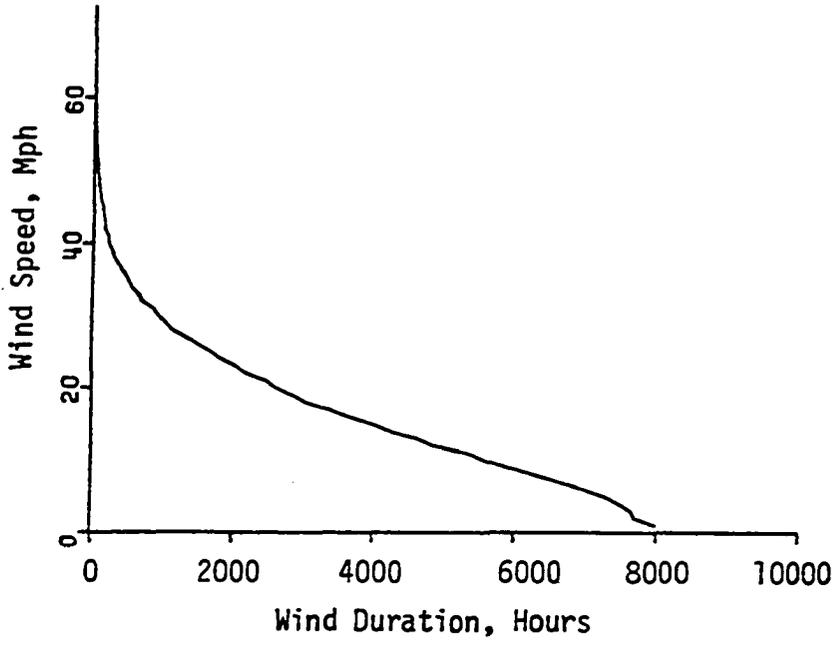


Fig. 3-4. Wind Duration for Station 3

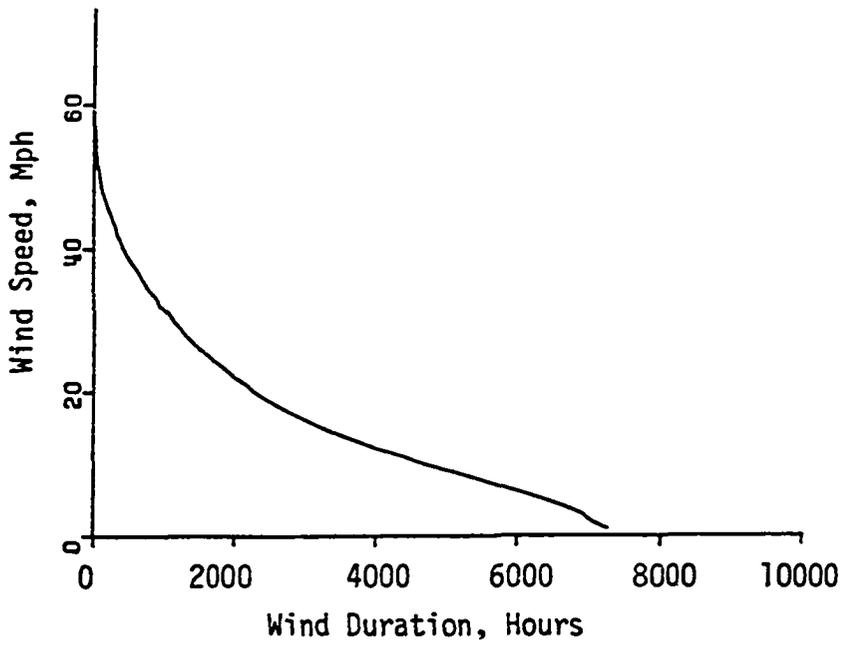


Fig. 3-5. Wind Duration for Station 4

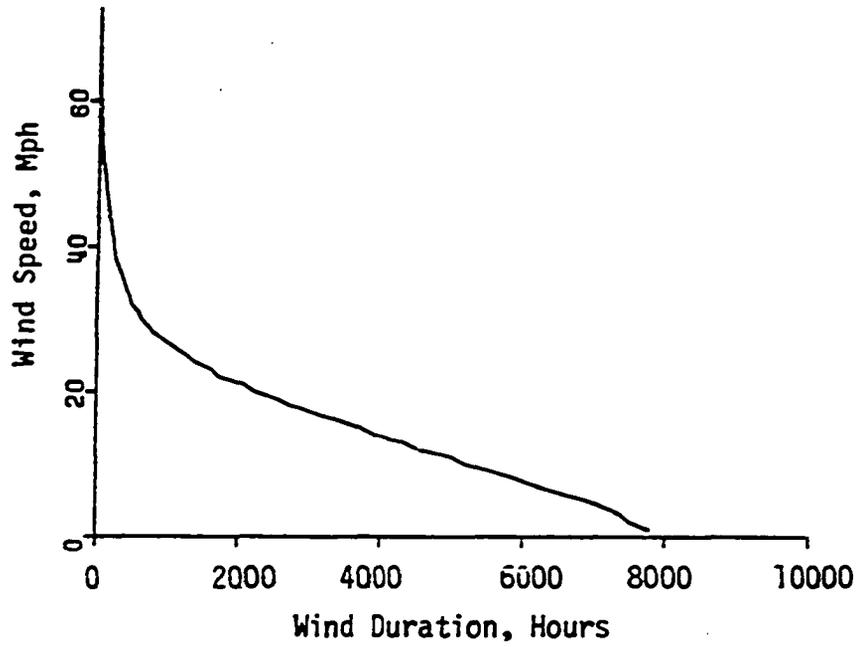


Fig. 3-6. Wind Duration for Station 5

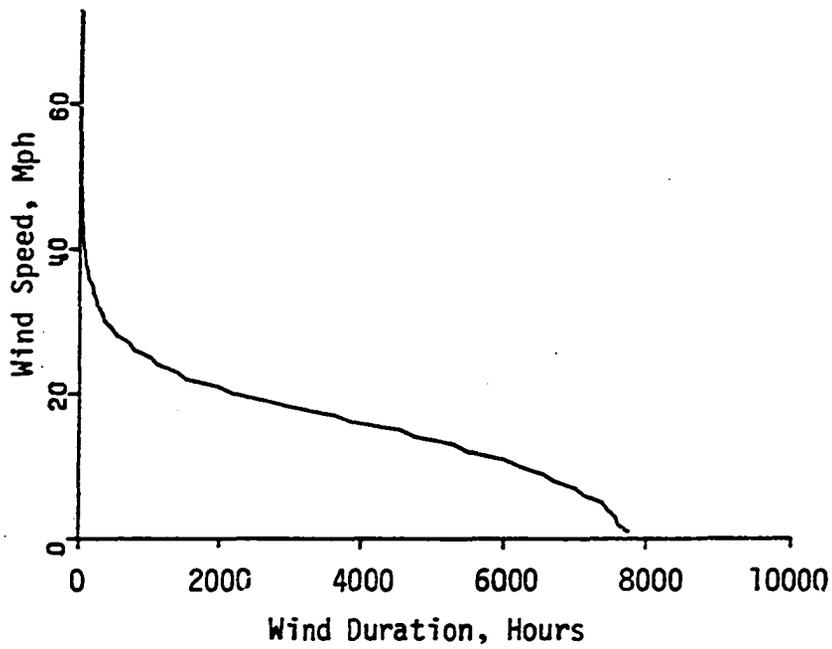


Fig. 3-7. Wind Duration for Station 6

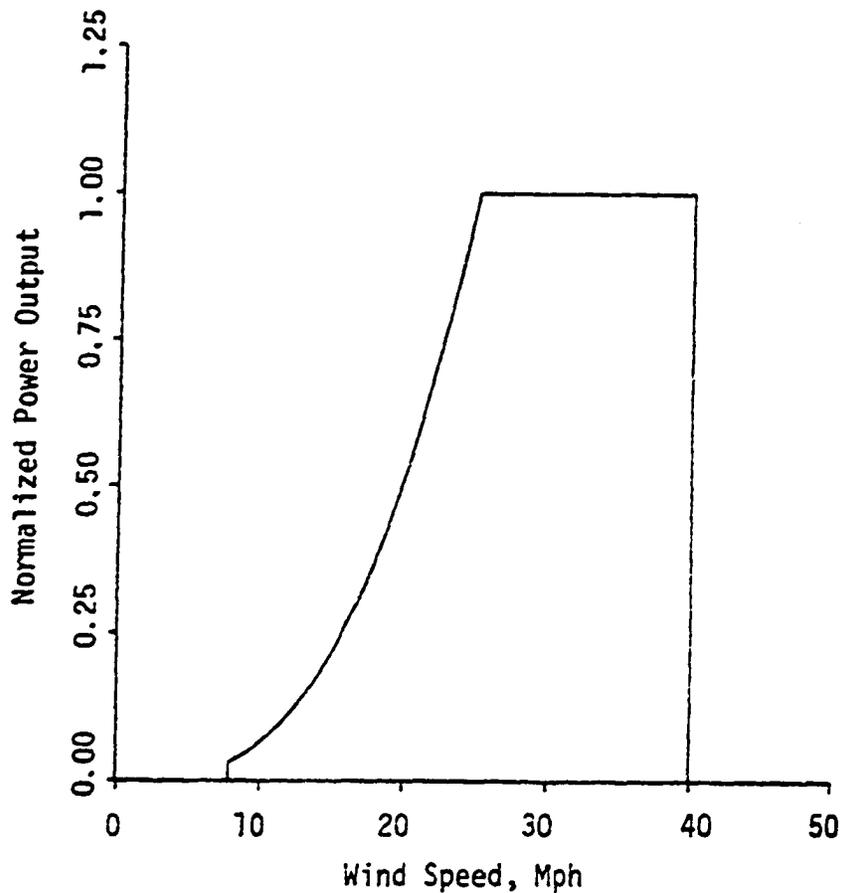


Fig. 3-8. Wind Turbine Generator Power Limits Used for This Study

Fig. 3-8. Although there is no electrical power produced for wind speeds below 8 mph or above 40 mph, this causes only a relatively small energy loss as indicated by the speed duration curves, Figs. 3-2 to 3-7.

The energy in the wind for speeds less than eight mph is small due to the cubing of wind speeds needed to get power. However, the eight mph low limit does cause substantial idle time for the individual stations. For example, Fig. 3-5 shows that station 4 would be idle for

approximately 3707 hours, or 42 percent of the time during the year. A comparison of the duration curves shows that station 6 would have the least idle time at 2216 hours for the year, which is still 25 percent of the time. Such substantial idle times prevent wind generators at a single site from contributing toward firm generating capacity, although there can be considerable benefit from the energy production alone. The improvement toward firm generating capacity that can be obtained by combining the power from several stations into a network power is examined in Chapter 4. Substantial improvement is possible over that for an individual station.

The energy lost due to the saturation speed, in this case 25 mph, and the shut-down speed, at 40 mph, can be quite large. These limits can be raised with a corresponding increase in the installed cost of a wind turbine generator, but economic considerations prevent installing a significantly over-sized unit in order to capture the relatively short duration energy in the higher wind speed.

The wind speeds exceeded 40 mph at Station 4 for the greatest time duration, which was only 4 percent of the time. All other stations had high winds less than that amount, and Station 1 had no wind speeds in excess of 40 mph. The saturation level which produced rated power between 25 and 40 mph occurred a maximum of 23 percent of the time for Station 2, 17 percent for Station 3 and 4, and a minimum of 4 percent for Station 1. Thus, using a shut-down speed of 40 mph does not greatly increase the total down time at each station, which is primarily caused by low wind speeds. The rated wind speed, in this case 25 mph, does cause an energy

loss and the optimal rated speed would depend on wind generator costs, the value of the generated electrical power, and electrical substation costs.

Mean Values

The annual mean values of wind speeds for each station are listed in Table 3-1. Station 2 has the greatest mean at 16.3 mph, and Station 1 has the lowest at 12.8 mph. Due to power being proportional to the cube of wind speed, seemingly small differences in mean wind speeds can be significant. The means of the wind speeds cubed and usable wind power densities are also listed in Table 3-1. These are used in the network power analysis presented in Chapter 4. The power quantities, which are the cubed speeds and power densities, are for usable wind power as defined by the limits given in Fig. 3-8. Thus, wind speeds greater than 40 mph have been removed and wind speeds between 25 and 40 mph have been treated as if they are all at 25 mph. This approach gives realistic power values for wind generator network analysis.

Table 3-1. Annual Mean Values for each Station

Station	Mean Wind Speed, mph	Mean of Cubed Speeds, (mph) ³	Wind Power Density	
			w/ft ²	w/m ²
1	12.8	3560	18.5	199
2	16.3	5771	30.0	323
3	15.4	4930	25.6	276
4	15.3	4505	23.4	252
5	14.3	4280	22.3	240
6	15.1	4625	24.0	259

CHAPTER 4.

ANALYSIS OF A POSSIBLE WIND GENERATOR NETWORK

To demonstrate the potential of wind power generation and the methods of analysis for a possible wind generator network, the wind data from the six sites shown in Fig. 3-1 were used. The cube of the hourly wind speed was used primarily in the computations, since wind power (and electrical generated power) is proportional to wind speed cubed. Optimum allocations are determined for a given number of large wind generator units among selected sites and the resulting wind generator network power is examined. The optimum allocations, or weightings, are based on the methods of Chapter 2. Annual and monthly performance results are presented for various wind generator network configurations, composed of subsets of the six sites discussed in Chapter 3.

Various statistical quantities must be calculated from the wind data. The hourly values of wind speeds at the six sites are used to determine the monthly and annual wind power means and variances, as well as the covariances among all the sites. Weightings (i.e., allocations of wind generators) are computed based on the annual statistics which minimize the annual network power variance.

For purposes of comparison, the monthly statistics can be used to compute monthly optimal weightings and the corresponding monthly network variances. Of course, the actual allocations of wind generators among several sites would be fixed from month to month, and so would be based on some desired annual average statistics. However, the calculation of the monthly optimal performance gives an ideal that is useful for

assessing the month-to-month performance for the weighting based on the annual statistics.

Network variance is used herein as a convenient measure of the network power fluctuations, and it is determined for both annual and monthly performances using a variety of weighting schemes. The weightings are scaled to yield a desired average network power, which is arbitrarily chosen to be unity.

Annual Network Performance

A measure of the performance of the wind generator network can be obtained by computing the variances of the network power production. This was done using different combinations of the six example sites, with variances computed for the year. Results are presented later that show some of the monthly variations.

For consistency, the weightings for the sites being used were chosen such that

$$\underline{w}^T \underline{x} = 1$$

which causes the average network power to be constant. Then, the individual weightings w_i were determined so that the network variance was minimized.

Table 4-1 shows the resulting annual variance of wind power for each station individually. This is the variance that would occur if the network consisted of only one station. A comparison of later results for multi-station networks with these single-station variances will show the advantage due to wind generator diversity over a wide geographical area. For the same power, Table 4-1 shows that there is over a 50 percent

increase in variance from the minimum variance site (no. 2) to the

Table 4-1. Annual Variances for Individual Stations

Variance	1.21	1.02	1.29	1.58	1.47	1.04
Station	1	2	3	4	5	6

maximum variance site (no. 4).

When two or more sites are combined, the resulting network variance will depend on the individual station variances as well as the correlations among the different sites in the network. Table 4-2 shows the correlation matrix for the year for the six sites. Most sites tend to be positively correlated with other sites, with the greatest correlation

Table 4-2. Correlation Matrix for the Year

	1	1.00	0.35	0.27	0.25	-0.00	0.46
	2		1.00	0.19	0.28	-0.08	0.33
	3			1.00	0.10	0.02	0.12
	4				1.00	-0.02	0.14
	5					1.00	0.01
	6						1.00
		1	2	3	4	5	6
Station							

occurring between sites 1 and 6. Site 5 tends to be slightly negatively correlated with most other sites; the greatest negative correlation

exists between sites 2 and 5. The site-to-site correlations affect the network variances presented below.

Table 4-3 shows the annual variances for all of the two station networks that are possible from the 6 sites being considered. The lowest

Table 4-3. Annual Variances for Two-Station Network

6						
5					.61	
4				.75	.71	
3			.78	.70	.64	
2		.68	.79	.55	.68	
1	.74	.79	.85	.66	.81	
	i	2	3	4	5	6
		Network Station				

variance of 0.55 exists for sites 2 and 5, while the greatest variance is 0.85 for sites 1 and 4. Note that the lowest variance does not result by selecting the two sites with the lowest individual variances, which from Table 4-1 are sites 2 and 6. Similarly, the two sites with greatest individual variances, site 4 and 5, give a network variance of 0.75, which is less than the maximum two-site network variance of 0.85. The two-site network reduces the minimum variance from the one-site system from 1.02 to 0.55, a reduction of 46 percent.

The three-site network variances are shown in Table 4-4. The maximum and minimum variances are 0.65 for sites 1, 2, and 4 and 0.44 for sites 2, 3, and 5, respectively. It is interesting that station 2

occurs in both groupings, which is due to the correlations among the different sites.

Table 4-4. Annual Variances for Three-Station Network

Network Station	6											.48		
	5											.53	.45	.46
	4			.53	.64			.48	.58	.51			.52	
	3			.63	.52	.60	.58	.44					.53	
	2	.59	.65	.47									.63	
		1,3	1,4	1,5	1,6	2,4	2,5	2,6	3,5	3,6	4,5			
Network Station														

Shown in Table 4-5 are the annual variances for the four-site networks. The minimum variance network is for sites 2, 3, 5 and 6 and is 0.38, while the maximum variance network is for sites 1, 2, 4, and 6 and is 0.56. Note that in Tables 4-5 and 4-6, the variances are identified by the sites omitted, rather than sites included, in the network.

Table 4-5. Annual Variances for Four-Station Network

Station Skipped	5						.53	
	4					.51	.41	
	3				.42	.56	.43	
	2			.45	.43	.51	.44	
	1	.39	.40	.38	.47			.40
		2	3	4	5	6		
Station Skipped								

The variances for a five-station network, shown in Table 4-6, range from a minimum of 0.35 for sites 2, 3, 4, 5 and 6 to a maximum of 0.47 for sites 1, 2, 3, 4 and 6. The network variance resulting from including all six sites in the network is 0.35, which is the same (to two decimal places) as the minimum variance for the five-site network.

Table 4-6. Annual Variances for Five-Station Network

Variance	.35	.38	.39	.37	.47	.38
Station Skipped	1	2	3	4	5	6

A summary of the variances for different numbers of stations in the network is presented in Fig. 4-1. Maximum and minimum variances for each network configuration are plotted along with the corresponding station groupings. There is a large reduction in network variance from the one-site network to the six-site network, which shows the improvement possible by locating wind generator sites over a wide geographical area. As expected, the relative improvement decreases as sites are added. There is a 14 percent reduction in the variance by adding a fourth site to the three site network, while there is only an 8 percent reduction by expanding the four-station network to six stations. This suggests a limiting beneficial effect from diversification as the number of sites is increased. For the set of six sites analyzed herein, a network of the four "best" sites may be sufficient, rather than developing one or two additional sites for a somewhat limited benefit. Of course, factors such as available land area, need for additional wind generated power, and location of the

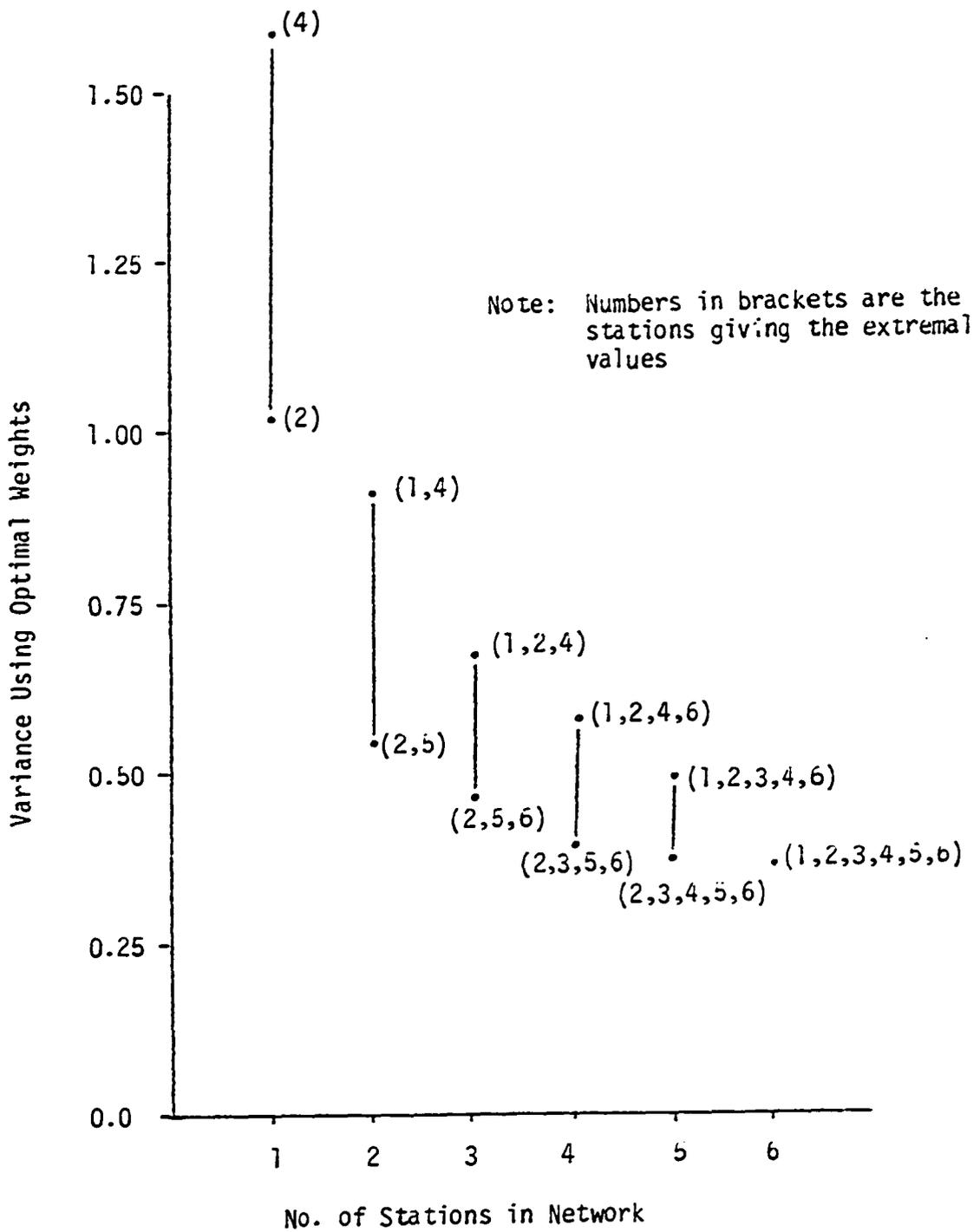


Fig. 4-1. Variance Extremes for Different Number of Stations in Network

additional sites relative to electrical loads and transmission facilities would also influence a decision as to whether more sites would be developed.

The variances in Fig. 4-1 show that a 66 percent reduction in network variance is possible by expanding from one to six stations. This reduced fluctuation, while producing the same average power, increases the tendency of the wind generator network for firm generating capacity. This aspect is explored in the next section.

Wind Power Duration

Using the site weightings computed from annual means and covariances among the six stations, wind power duration curves were determined for the best (minimum variance) network configurations. These groupings of the stations are those in Fig. 4-1 which resulted in the minimum variance for each total number of network stations from one to six. The corresponding wind power duration curves are shown in Figs. 4-2 through 4-7.

The various configuration weightings were all constrained to produce the same annual average power. Thus, the curves can be compared to determine differences in the duration of the power, with particular interest in any improvement at the lower power levels. All sites have a certain amount of substantial wind power, but by combining the powers from various sites, there can be a reduction in the length of time of little or no useful wind power. This will, of course, be at the expense of somewhat reduced maximum network power from that for a single site producing the same average power; however, this does not create a problem and is a desirable tradeoff.

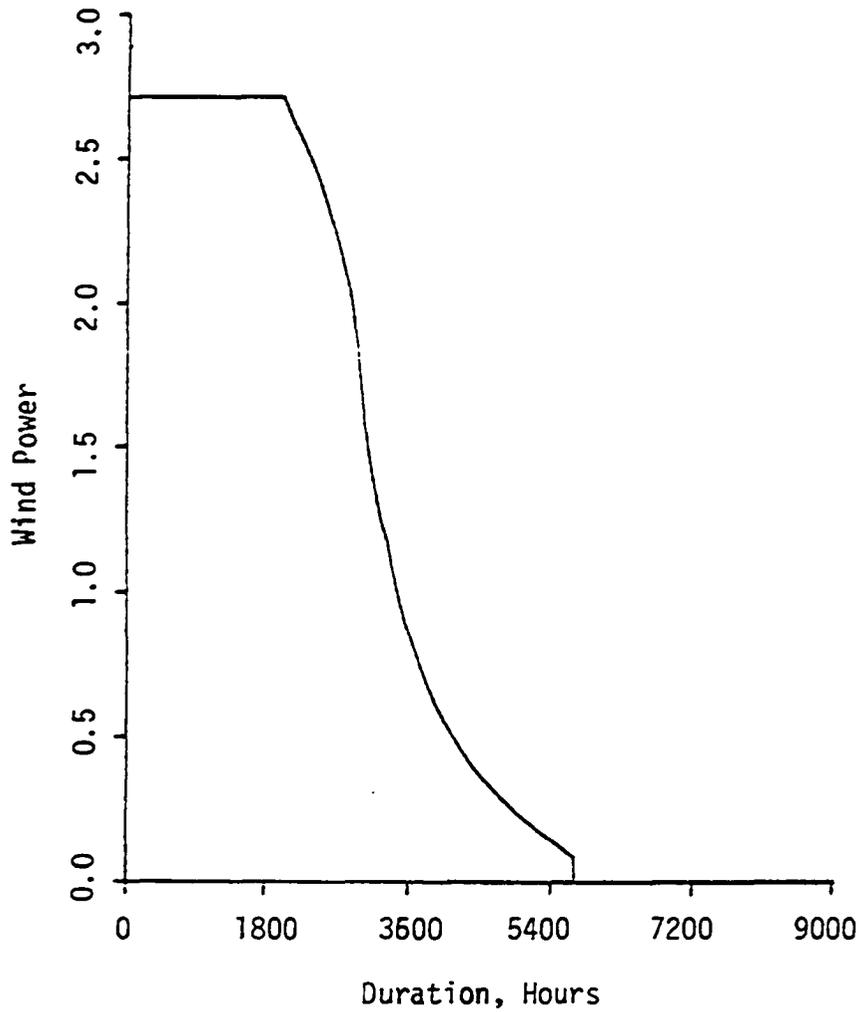


Fig. 4-2. Annual Wind Power Duration for the One-Site Network

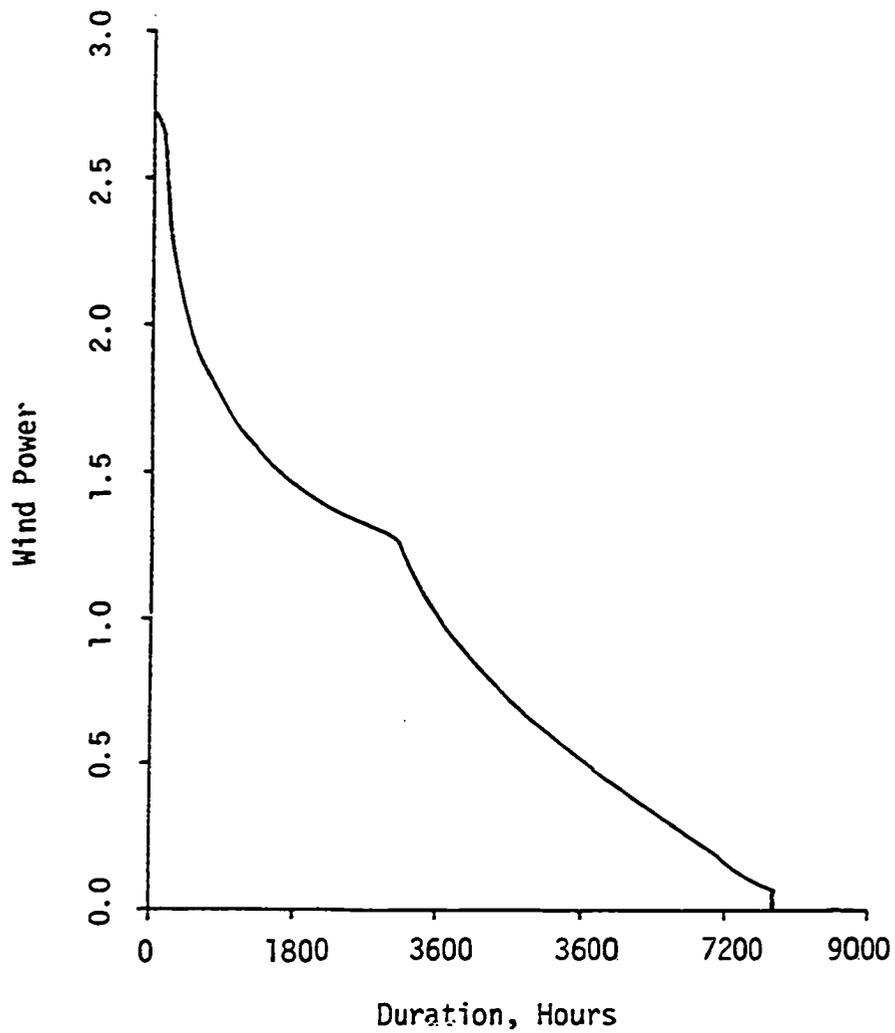


Fig. 4-3. Annual Wind Power Duration for the Two-Site Network

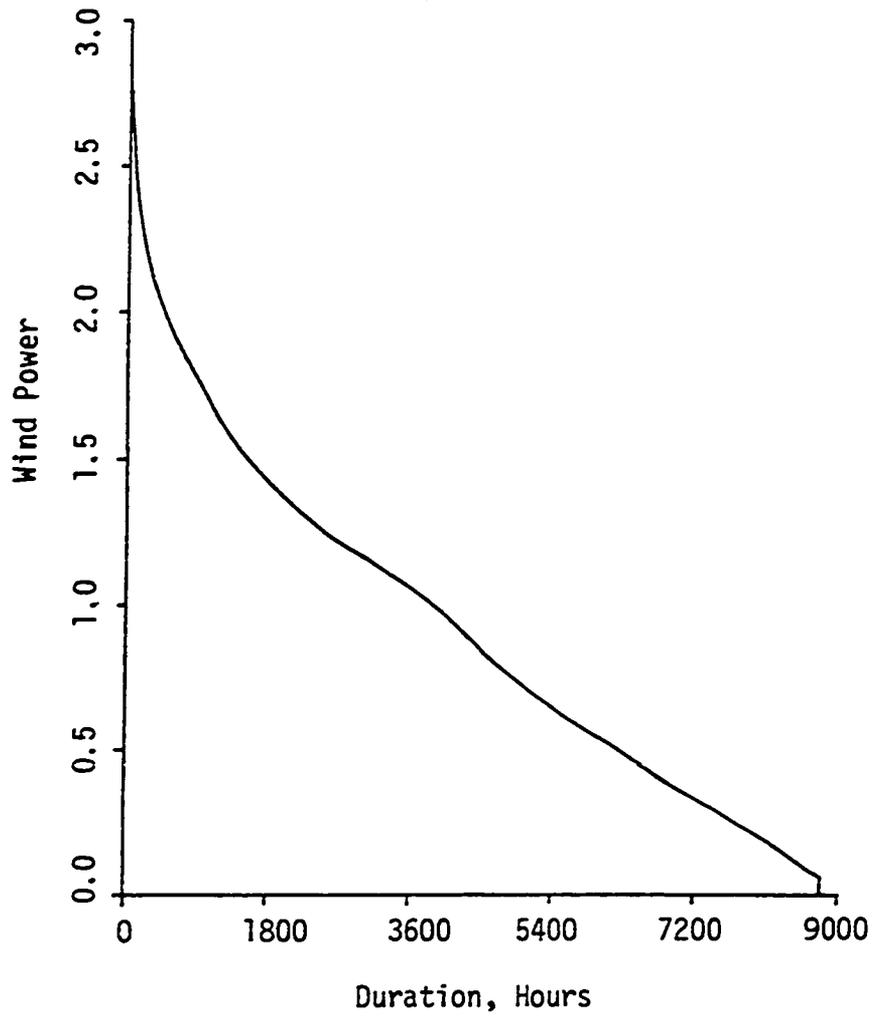


Fig. 4-4. Annual Wind Power Duration for the Three-Site Network

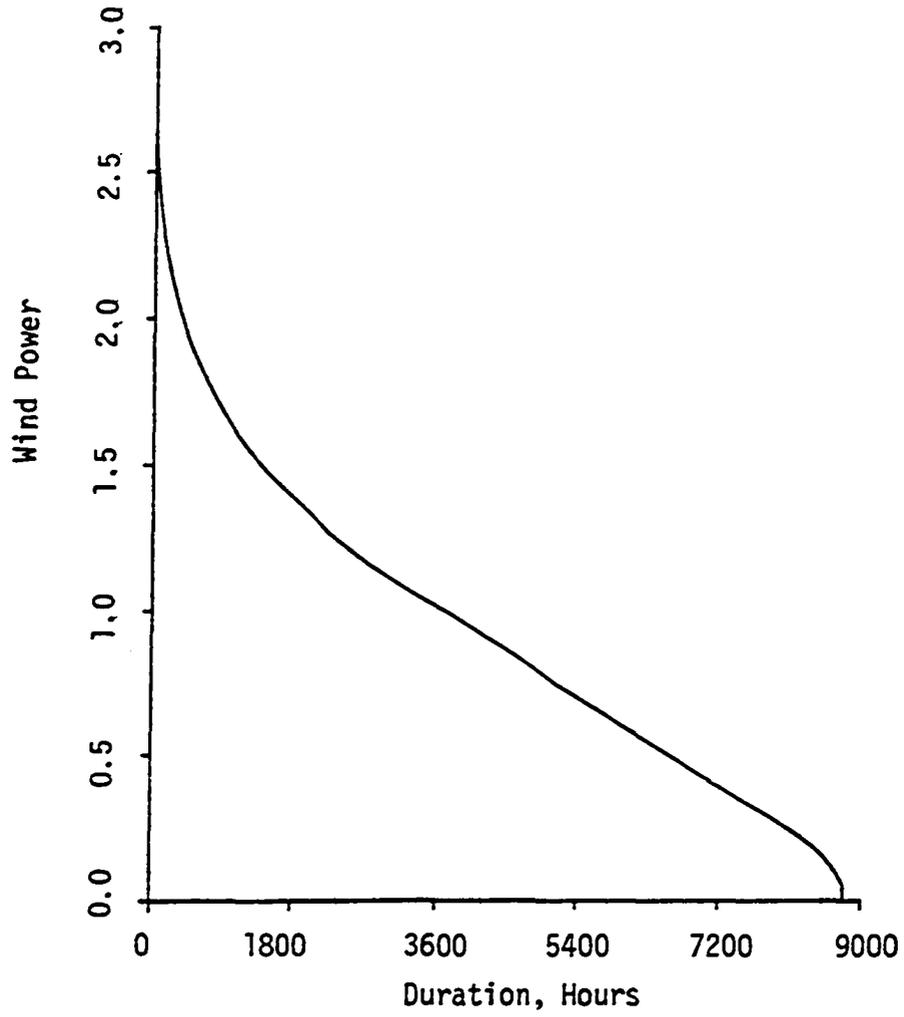


Fig. 4-5. Annual Wind Power Duration for the Four-Site Network

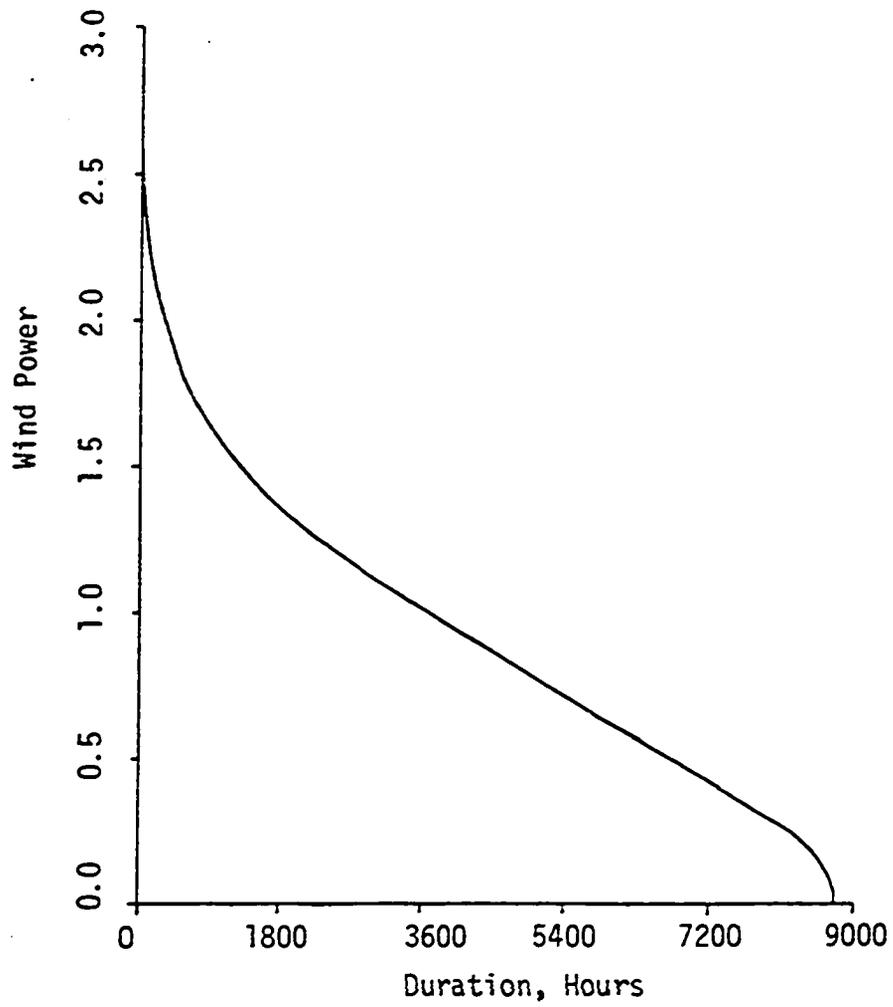


Fig. 4-6. Annual Wind Power Duration for the Five-Site Network

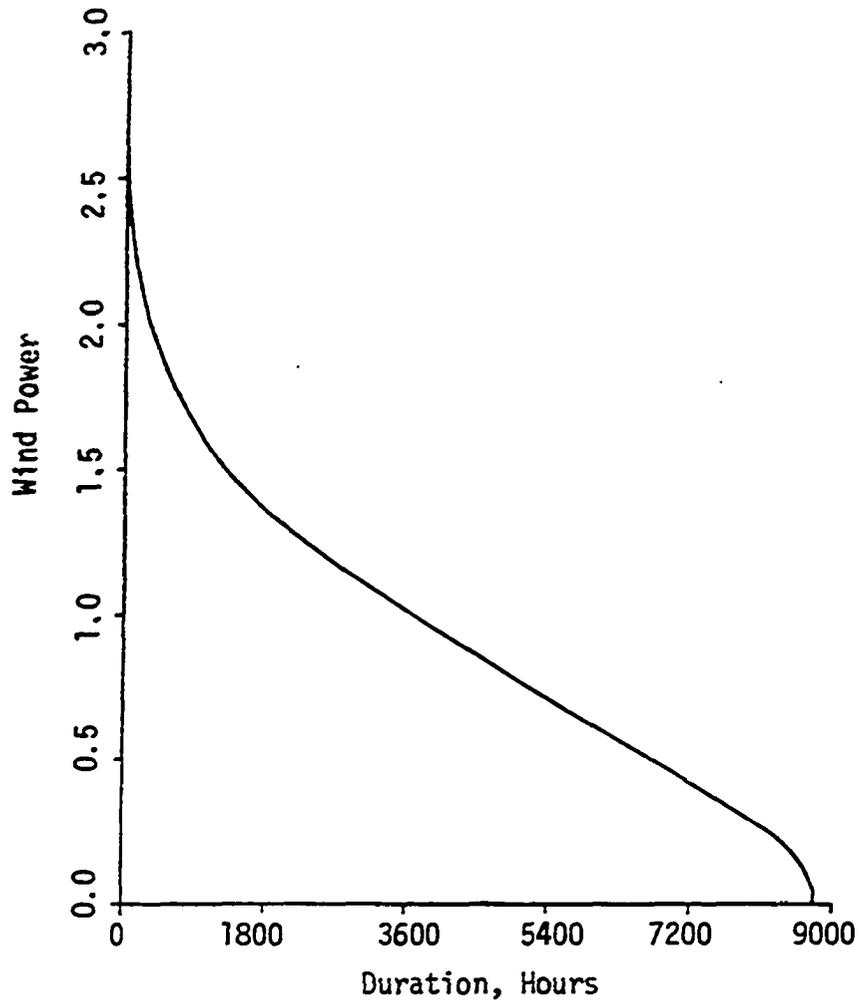


Fig. 4-7. Annual Wind Power Duration for the Six-Site Network

The minimum annual variance site is station two whose power duration curve is shown in Fig. 4-2. Although substantial power occurs for a portion of time, there is also a significant length of time during which there exists very little power. This curve shape is typical of many individual sites, which even may have reasonably high annual mean wind speeds. For consistent electrical generation, it is more desirable to have a power duration curve that has a "flatter" characteristic shape. Such a site or network may show little or no improvement in average power, but the duration of the power above a minimum level will be greater.

For the best two site network, which uses sites two and five, the power duration curve is shown in Fig. 4-3. The two sites complement each other so that the low power portion of the curve (toward the right) is substantially better than that for the one-site network shown in Fig. 4-2. For example, the power level equal to 20 percent of the average power is exceeded 84 percent of the time during the year for the two-site network, while that level is exceeded only 67 percent of the time for the one-site case.

The other network configurations, for groupings of three to six stations, show a similar trend. As the network is expanded, the power duration curve becomes flatter. The time during the year that a power level of 20 percent of the network mean power is exceeded are 92, 95, 96 and 97 percent respectively, for the network configurations comprised of three, four, five and six stations.

A comparison of Figs. 4-4 through 4-7 shows that there is a decreasing benefit as each additional station is added. It was noted

earlier that only a very small improvement in reduced variance was achieved by going from the five to the six-station network. This is supported by comparing the corresponding duration curves in Figs. 4-6 and 4-7; there is no substantial difference between them. There is significant benefit, however, for including up to four stations in the network. Additional stations beyond this number seem to add redundancy while not providing wind power that is significantly different statistically from that for the four-station network. In developing an actual wind generator network, it may be of benefit in a case like this to examine the wind power characteristics of additional potential sites so as to expand the network with essentially statistically independent (or negatively correlated) wind power. However, from the practical side, it is a case of diminishing return, and extensive further site investigation may not be justified.

In order to more easily compare the durations of various power levels for the different network configurations, the percentages are listed in Table 4-7 for the time that the specified power levels are exceeded. At the low power levels, there is a considerable improvement in the durations by employing multiple-site configurations. At power levels of 0.6 and 0.8 of the mean, there is still improvement from 61 to 70 percent and 52 to 57 percent, respectively, in going from a two-site network to four sites. At the power level equal to the mean power, there is little change in the time duration for the different numbers of sites, except for expanding from the single site to two sites. At power levels greater than the mean, the time duration decreases as more sites are

Table 4-7. Percent of Time that Power Level is Exceeded

Power Level (fraction of mean power)	Number of Stations in Network					
	6	5	4	3	2	1
2.0	4%	4%	5%	8%	7%	23%
1.8	7%	8%	8%	11%	12%	24%
1.6	13%	13%	14%	16%	19%	27%
1.4	21%	21%	22%	23%	33%	30%
1.2	31%	31%	31%	34%	39%	34%
1.0	45%	45%	44%	46%	44%	35%
0.8	58%	58%	57%	55%	52%	44%
0.6	71%	71%	70%	66%	61%	50%
0.4	85%	85%	83%	78%	72%	58%
0.2	97%	96%	95%	92%	84%	67%

are included in the network. This is caused by the same influences that produce improvements in the duration at low power levels. By combining stations in a network, the power level extremes are moderated.

Monthly Wind Power

In the previous section, the results show the annual performance of the network configurations. The weightings were calculated using the wind power statistics for the year, and the site weightings did not vary from month to month. This is similar to an actual wind generator network in which a specified number of wind generators are installed at each site. It is interesting, however, to compare the monthly performance using the "annual" weightings with what could be achieved if weightings were used

that minimized the network variance for that month. This comparison will show how well the annual weightings perform during each month.

The month-by-month analysis suggests an alternative approach to the annual weighting. Instead of basing the weightings on all 12 months of wind data, the weightings could be based on certain selected months or seasons. This would cause the wind network performance during that time period to be improved over that for using weightings based on the full year of data. Of course, this approach will tend to degrade performance during those months not included in the weighting calculation. A generalization of this technique is to apply arbitrary monthly weightings to the wind data prior to computing the annual statistics. Then the resulting wind site weightings would reflect, on the average, the desire to minimize variances during those months with the greatest weightings. This approach is developed in the following.

Let P_i , $i = 1, 2, \dots, 12$, be the monthly covariance matrices for the network sites. Similarly, let \underline{m}_i , $i = 1, 2, \dots, 12$, be the monthly means. Then equivalent annual average values P and \underline{m} are computed using arbitrary weightings of the monthly values. This gives the following:

$$P = \frac{1}{f} \sum_{i=1}^{12} s_i d_i P_i \quad 4-1$$

$$\underline{m} = \frac{1}{f} \sum_{i=1}^{12} s_i d_i \underline{m}_i \quad 4-2$$

where

$$f = \sum_{i=1}^{12} s_i d_i \quad 4-3$$

and d_i is the number of days in the i th month and s_i is the arbitrary monthly weighting. One choice of values for s_i is simply zeros and ones; zeros are applied to those months for which minimizing the variance is not crucial, perhaps during one particular season. The site weightings \underline{w} are then calculated from \underline{P} and \underline{m} using the minimum-variance method presented in Chapter 2.

Monthly wind power statistics were calculated from the wind data for the six sites. For convenience in showing the site-to-site relationship, the monthly correlation matrices were calculated. These matrices, along with the variances and means, are listed in Table 4-8 for each month of the year.

The site-to-site correlations vary over a wide range throughout the year. There are a number of correlations in excess of 0.5, many in the vicinity of zero, and some as negative as -0.2 and -0.3. As an example, consider the correlations between sites one and four. Initially the correlation is about 0.2, but then dips to below -0.1 in month three. In months six and seven the wind power deviations are more strongly correlated from those two sites with values of 0.6 and 0.4, respectively, while during the remainder of the year the correlation is around 0.2 to 0.3. Such changes in the correlations, as well as similar monthly changes in the means and variances, preclude the weighting based on annual averages from being optimal for each month. In the following discussion the effects of the monthly variations are analyzed.

Using all twelve months with s_i , $i = 1, 2, \dots, 12$, equal to one in eqs. 4-1 and 4-2, the optimal site weightings were determined for the

Table 4-8. Monthly Correlation Matrices, Variances, and Means of Wind Speeds Cubed

Correlation Matrix						Variances (mph) ⁶ × 10 ⁶	Means (mph) ³
(a) Correlations, Variances, and Means for Month 1							
1.000	0.482	-0.007	0.195	-0.137	0.525	5.47	1557
0.482	1.000	0.179	0.519	-0.156	0.229	13.7	1756
-0.007	0.179	1.000	0.188	-0.110	-0.026	40.5	6451
0.195	0.519	0.188	1.000	-1.181	-0.005	20.9	2119
-0.137	-0.156	-0.110	-0.181	1.000	-0.272	30.3	4065
0.525	0.229	-0.026	-0.005	-0.272	1.000	7.53	2582
(b) Correlations, Variances and Means for Month 2							
1.000	0.174	0.333	0.220	-0.015	0.045	16.2	3304
0.174	1.000	0.400	0.061	0.113	-0.152	32.8	4869
0.333	0.400	1.000	0.279	-0.108	0.198	41.7	6399
0.220	0.061	0.279	1.000	0.051	0.004	45.8	4909
-0.015	0.113	-0.108	0.051	1.000	-0.048	38.1	5818
0.045	-0.152	0.198	0.004	-0.048	1.000	14.7	2871
(c) Correlations, Variances and Means for Month 3							
1.000	0.068	0.209	-0.117	0.217	0.446	25.6	5564
0.068	1.000	0.180	0.081	-0.019	0.383	40.5	9183
0.209	0.180	1.000	-0.106	-0.152	0.027	45.3	7670
-0.117	0.081	-0.106	1.000	-0.099	0.017	39.4	6853
0.217	-0.019	-0.152	-0.099	1.000	0.036	31.0	4748
0.466	0.383	0.027	0.017	0.036	1.000	27.0	5595
(d) Correlations, Variances and Means for Month 4							
1.000	0.249	0.225	0.239	0.175	0.555	14.0	3510
0.249	1.000	0.290	0.199	-0.013	0.555	36.5	5282
0.225	0.290	1.000	0.025	0.251	0.237	31.5	5271
0.239	0.199	0.025	1.000	-0.063	0.186	30.6	4279
0.175	-0.013	0.251	-0.063	1.000	0.293	36.0	5843
0.555	0.555	0.237	0.186	0.293	1.000	23.2	4636

Table 4-8 (continued) Monthly Correlation Matrices, Variances, and Means of Wind Speeds Cubed

(e) Correlations, Variances and Means for Month 5

1.000	0.447	0.599	0.096	-0.185	0.705	18.8	4831
0.447	1.000	0.358	0.228	-0.157	0.325	42.2	8717
0.599	0.358	1.000	-0.179	-0.090	0.277	28.0	4415
0.096	0.228	-0.179	1.000	-0.005	0.176	43.6	6553
-0.185	-0.157	-0.090	-0.005	1.000	-0.065	23.6	3471
0.705	0.325	0.277	0.176	-0.065	1.000	26.0	5294

(f) Correlations, Variances, and Means for Month 6

1.000	0.346	0.244	0.645	-0.054	0.439	24.2	5706
0.346	1.000	0.112	0.164	-0.109	0.207	42.1	8951
0.244	0.112	1.000	0.066	-0.067	0.136	37.8	7006
0.645	0.164	0.066	1.000	0.580	0.450	31.7	4927
-0.054	-0.109	-0.067	0.580	1.000	-0.024	33.9	5729
0.439	0.207	0.136	0.450	-0.024	1.000	21.7	4639

(g) Correlations, Variances, and Means for Month 7

1.000	0.622	0.416	0.436	-0.004	0.440	19.9	4647
0.622	1.000	0.105	0.347	-0.196	0.284	38.5	8246
0.416	0.105	1.000	0.489	0.437	0.213	24.0	3915
0.436	0.347	0.489	1.000	-0.092	-0.191	28.6	4411
-0.004	-0.196	0.437	-0.092	1.000	0.150	24.5	4021
0.440	0.284	0.213	-0.191	0.150	1.000	37.1	6066

(h) Correlations, Variances, and Means for Month 8

1.000	0.631	0.386	0.334	-0.276	0.237	10.2	2919
0.631	1.000	0.101	0.373	-0.358	0.425	36.9	6447
0.386	0.101	1.000	0.083	0.126	0.082	21.6	3411
0.334	0.373	0.083	1.000	-0.383	-0.103	35.1	5261
-0.276	-0.358	0.126	-0.383	1.000	-0.103	19.4	3633
0.237	0.425	0.082	-0.103	-0.103	1.000	19.3	3709

Table 4-8 (continued) Monthly Correlation Matrices, Variances, and Means of Wind Speeds Cubed

(i) Correlations, Variances, and Means for Month 9

1.000	0.475	0.317	0.368	0.088	0.269	13.4	2944
0.475	1.000	0.174	0.608	-0.152	0.479	34.6	5142
0.317	0.174	1.000	0.201	-0.015	0.228	19.7	2798
0.368	0.608	0.201	1.000	-0.156	0.489	27.5	3681
0.088	-0.152	-0.015	-0.156	1.000	0.010	16.9	2904
0.629	0.479	0.228	0.489	0.010	1.000	26.9	5403

(j) Correlations, Variances, and Means for Month 10

1.000	0.346	0.321	0.293	-0.018	0.564	14.7	3495
0.346	1.000	0.103	0.274	0.044	0.488	31.2	4311
0.321	0.103	1.000	0.160	-0.059	0.137	25.8	3587
0.293	0.274	0.160	1.000	0.014	0.351	27.5	3397
-0.018	0.044	-0.059	0.014	1.000	0.041	14.8	2482
0.564	0.488	0.137	0.351	0.041	1.000	27.3	5648

(k) Correlations, Variances, and Means for Month 11

1.000	0.538	0.154	0.253	0.000	0.565	9.58	2202
0.538	1.000	0.155	0.511	-0.070	0.410	26.8	2836
0.154	0.155	1.000	0.157	0.118	0.011	33.0	4257
0.253	0.511	0.157	1.000	-0.009	0.205	20.8	2699
0.000	-0.070	0.118	-0.009	1.000	-0.037	33.2	4843
0.565	0.410	0.011	0.205	-0.037	1.000	18.2	5061

(l) Correlations, Variances, and Means for Month 12

1.000	0.006	0.050	0.205	0.027	0.281	11.6	2016
0.006	1.000	0.138	0.316	0.050	0.189	30.5	3394
0.050	0.138	1.000	0.080	0.065	-0.058	28.3	4113
0.205	0.316	0.080	1.000	-0.026	0.141	34.4	4027
0.027	0.050	0.065	-0.026	1.000	-0.106	23.1	4026
0.281	0.189	-0.058	0.141	-0.106	1.000	16.3	3861

different network configurations. In addition, the optimal site weightings were determined for each month individually. Actually, two "optimal" monthly weightings were determined, one with unconstrained weights and the other with each weight constrained to be non-negative. The constrained weights are the optimal from a physically realizable viewpoint, while the unconstrained weights can be used to determine the effect of the non-negative constraint. The three sets of weightings for the six-site network are listed in Table 4-9. The weightings have been normalized for ease of comparison; each weighting listed is the fraction of the total number of wind generators allocated to a site.

If an n-site network has fewer than n non-zero weights, then the minimum variance criterion requires that no wind generators be allocated to one or more sites. This can happen if an unconstrained weighting is negative, and then constraining the weights to be non-negative can force that weighting to be zero. This will tend to occur for a site that is highly correlated to another network site.

As shown in Table 4-9, occasionally, one or more of the unconstrained monthly weighting values are negative. Thus, it was necessary to apply the non-negative constraint on the weightings and to solve for the physically realizable optimal weights using numerical mathematical programming techniques. In each instance in which the unconstrained weight is negative, the corresponding constrained weight is zero. Also the remaining weights for that month are altered somewhat to compensate. For example, in month seven there are two negative unconstrained weights, -0.1032 and -0.0806 for sites one and three, respectively.

Table 4-9. Monthly and Annual Normalized Weightings,
Six-Station Network

Optimal Weightings for each Month (unconstrained)

1.	0.0588	-0.437	0.1570	0.1092	0.2121	0.4193
2.	0.1520	0.1460	0.0842	0.1201	0.2109	0.2869
3.	0.1241	0.1726	0.1883	0.2310	0.2085	0.0754
4.	0.2080	0.1649	0.1519	0.2011	0.2500	-0.0192
5.	0.1045	0.2043	0.1290	0.1749	0.2953	0.0920
6.	0.2228	0.0806	0.0534	-0.2695	0.2559	0.1177
7.	-0.1032	0.2057	-0.0806	0.2053	0.2451	0.1602
8.	0.0884	0.1088	0.0380	0.2098	0.3786	0.1763
9.	-0.0766	0.1753	0.1540	0.0292	0.3376	0.2272
10.	0.1193	0.0760	0.1902	0.0662	0.2098	0.2385
11.	-0.0549	-0.0328	0.1589	0.1088	0.2082	0.4364
12.	0.0733	0.0395	0.1875	0.1048	0.2621	0.3329

Constrained Weightings for each Month (non-negative)

1.	0.0355	0.0	0.1695	0.1035	0.2331	0.4584
2.	0.1520	0.1460	0.0842	0.1201	0.2109	0.2869
3.	0.1241	0.1726	0.1883	0.2310	0.2085	0.0754
4.	0.2062	0.1642	0.1590	0.2091	0.2616	0.0
5.	0.1045	0.2043	0.1290	0.1749	0.2953	0.0920
6.	0.1056	0.2513	0.2094	0.0	0.2874	0.1463
7.	0.0	0.2933	0.0	0.2013	0.3288	0.1766
8.	0.0884	0.1088	0.0380	0.2098	0.3786	0.1763
9.	0.0	0.1878	0.1649	0.0362	0.3796	0.2315
10.	0.1193	0.0760	0.1902	0.0662	0.3098	0.2385
11.	0.0	0.0	0.1787	0.1034	0.2462	0.4717
12.	0.0733	0.0395	0.1875	0.1048	0.2621	0.3329

Optimal Weighting for the Year

0.0709	0.1582	0.1708	0.1342	0.2695	0.1965
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The constrained weights for those sites are zero, and the remaining weights for sites two, four, five and six change from 0.21 to 0.29, 0.21 to 0.20, 0.25 to 0.33 and 0.16 to 0.18, respectively. Computation of the annual weighting did not require applying the non-negative constraint.

The monthly variances for the six-site network are shown in Table 4-10. The monthly weightings, both constrained and unconstrained, are

Table 4-10. Monthly Performance for Six-Site Network

Month	Network Power Annual Weighting	Variance, Annual Weighting	Variance Monthly Non-Negative	Variance, Monthly Unconstrained
1	.723	.190	.146	.145
2	1.07	.378	.358	.358
3	1.38	.353	.318	.318
4	1.08	.492	.454	.454
5	1.14	.361	.346	.346
6	1.31	.446	.359	.263
7	1.11	.430	.352	.335
8	.904	.222	.192	.192
9	.821	.357	.332	.330
10	.808	.338	.323	.323
11	.854	.344	.275	.273
12	.806	.271	.240	.240

selected so as to produce a monthly average power of one. The annual weighting produces an annual average power of one, but the monthly average power, using the annual weighting, will vary. In order to compare the variances for the different weightings, the monthly

weightings were scaled so as to produce the same average monthly power as the annual weighting. The scale factor is simply the square of the monthly average power that resulted from using the annual weighting for that month, since the monthly optimal weightings are chosen so as to produce unity power.

Monthly weightings, annual weightings, and monthly network variances were computed for each of the network configurations. The format of the results for the six-station network, shown in Tables 4-9 and 4-10, is followed for the remaining network configurations in Tables 4-11 through 4-19. The stations comprising the various networks are the groupings shown in Fig. 4-1 which yielded the minimum annual variance.

The three monthly variances listed in Tables 4-10, -12, -14, -16, -18, are computed for the monthly average power listed in the column second from the left. That average monthly power resulted from applying the annual weighting to the network. For the months in which there are no negative, unconstrained weightings, the two right-hand columns will show the same variances. Otherwise, there will be a difference in those two columns, which can be considerable. See, for example, the variances for months six and seven in Table 4-10. The large difference between the variances for constrained and unconstrained weights for month six is likely due to the wind power from site four (negative unconstrained weighting) being highly correlated with a linear combination of the wind powers from the other sites. In such a case, the unconstrained minimization of the network variance will force a negative weighting to occur for that month, and constraining the weights to be non-negative

Table 4-11. Monthly and Annual Normalized Weightings,
Five-Station Network

 Optimal Weightings for each Month (unconstrained)

0.0	-0.0307	0.1639	0.1155	0.2245	0.4653
0.0	0.1703	0.1231	0.1488	0.2364	0.3214
0.0	0.1620	0.2196	0.2329	0.2427	0.1428
0.0	0.1691	0.1961	0.2572	0.2912	0.0863
0.0	0.2151	0.1679	0.1821	0.2919	0.1430
0.0	0.2036	0.1576	-0.1490	0.2841	0.2058
0.0	0.2095	-0.1167	0.2175	0.2959	0.1604
0.0	0.1412	0.0617	0.2232	0.3907	0.1831
0.0	0.1878	0.1649	0.0362	0.3796	0.2315
0.0	0.0861	0.2221	0.0758	0.3219	0.2940
0.0	-0.0480	0.1664	0.1175	0.2209	0.4473
0.0	0.0356	0.1998	0.1188	0.2783	0.3676

Constrained Weightings for each Month (non-negative)

0.0	0.0	0.1717	0.1090	0.2373	0.4820
0.0	0.1703	0.1231	0.1488	0.2364	0.3214
0.0	0.1620	0.2196	0.2329	0.2427	0.1428
0.0	0.1691	0.1961	0.2572	0.2912	0.0863
0.0	0.2151	0.1679	0.1821	0.2919	0.1430
0.0	0.2802	0.2307	0.0	0.2961	0.1929
0.0	0.2933	0.0	0.2013	0.3288	0.1766
0.0	0.1412	0.0617	0.2232	0.3907	0.1831
0.0	0.1878	0.1649	0.0362	0.3796	0.2315
0.0	0.0861	0.2221	0.0758	0.3219	0.2940
0.0	0.0	0.1787	0.1034	0.2462	0.4717
0.0	0.0356	0.1998	0.1188	0.2783	0.3676

Optimal Weighting for the Year

0.0	0.1699	0.1845	0.1446	0.2770	0.2241
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Table 4-12. Monthly Performance for the Five-Site Network

Month	Network Power, Annual Weighting	Variance, Annual Weighting	Variance, Monthly Weightings	Variance, Monthly Weighting (Unconstrained)
1	0.735	0.197	0.151	0.151
2	1.074	0.388	0.377	0.377
3	1.372	0.350	0.325	0.325
4	1.080	0.505	0.477	0.477
5	1.132	0.354	0.343	0.343
6	1.292	0.437	0.356	0.332
7	1.099	0.420	0.347	0.338
8	0.908	0.224	0.195	0.195
9	0.827	0.359	0.336	0.336
10	0.806	0.342	0.327	0.327
11	0.868	0.350	0.284	0.282
12	0.818	0.285	0.251	0.251

Table 4-13. Monthly and Annual Normalized Weightings,
Four-Station Network

Optimal Weightings for each Month (unconstrained)

0.0	0.0508	0.1979	0.0	0.2460	0.5054
0.0	0.1733	0.2000	0.0	0.2851	0.3417
0.0	0.2585	0.2598	0.0	0.2918	0.1899
0.0	0.2497	0.2426	0.0	0.3392	0.1685
0.0	0.3192	0.1100	0.0	0.3527	0.2181
0.0	0.2808	0.2307	0.0	0.2961	0.1929
0.0	0.4312	0.0574	0.0	0.3746	0.1368
0.0	0.3171	0.1285	0.0	0.4270	0.1273
0.0	0.2055	0.1698	0.0	0.3806	0.2411
0.0	0.0990	0.2395	0.0	0.3346	0.3269
0.0	0.0044	0.2049	0.0	0.2627	0.5280
0.0	0.0780	0.2208	0.0	0.2926	0.4091

Constrained Weightings for each Month (non-negative)

0.0	0.0508	0.1979	0.0	0.2460	0.5054
0.0	0.1733	0.2000	0.0	0.2851	0.3417
0.0	0.2585	0.2598	0.0	0.2918	0.1899
0.0	0.2497	0.2426	0.0	0.3392	0.1685
0.0	0.3192	0.1100	0.0	0.3527	0.2181
0.0	0.2802	0.2307	0.0	0.2961	0.1929
0.0	0.4313	0.0574	0.0	0.3746	0.1368
0.0	0.3171	0.1285	0.0	0.4270	0.1273
0.0	0.2055	0.1698	0.0	0.3806	0.2441
0.0	0.0990	0.2395	0.0	0.3346	0.3269
0.0	0.0044	0.2049	0.0	0.2627	0.5280
0.0	0.0780	0.2203	0.0	0.2926	0.4091

Optimal Weighting for the Year

0.0	0.2270	0.2114	0.0	0.3056	0.2560
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Table 4-14. Monthly Performance for the Four-Site Network

Month	Network Power, Annual Weighting	Variance, Annual Weighting	Variance, Monthly Weightings	Variance, Monthly Weighting (Unconstrained)
1	0.757	0.208	0.171	0.171
2	1.026	0.403	0.392	0.392
3	1.360	0.444	0.434	0.434
4	1.091	0.598	0.591	0.591
5	1.100	0.398	0.379	0.379
6	1.332	0.385	0.378	0.378
7	1.132	0.480	0.406	0.406
8	0.876	0.273	0.249	0.249
9	0.832	0.349	0.342	0.342
10	0.814	0.357	0.339	0.339
11	0.892	0.373	0.307	0.307
12	0.797	0.293	0.256	0.256

Table 4-15. Monthly and Annual Normalized Weightings,
Three-Station Network

Optimal Weightings for each Month (unconstrained)

0.0	0.1428	0.0	0.0	0.2781	0.5791
0.0	0.2877	0.0	0.0	0.2650	0.4473
0.0	0.4249	0.0	0.0	0.3351	0.2400
0.0	0.3587	0.0	0.0	0.4507	0.1907
0.0	0.3704	0.0	0.0	0.3740	0.2557
0.0	0.3682	0.0	0.0	0.3491	0.2827
0.0	0.4475	0.0	0.0	0.4084	0.1440
0.0	0.3623	0.0	0.0	0.4933	0.1444
0.0	0.2493	0.0	0.0	0.4379	0.3128
0.0	0.1411	0.0	0.0	0.4037	0.4552
0.0	0.0570	0.0	0.0	0.3410	0.6030
0.0	0.1376	0.0	0.0	0.3805	0.4819

Constrained Weightings for each Month (non-negative)

0.0	0.1428	0.0	0.0	0.2781	0.5791
0.0	0.2877	0.0	0.0	0.2650	0.4473
0.0	0.4249	0.0	0.0	0.3351	0.2400
0.0	0.3587	0.0	0.0	0.4507	0.1907
0.0	0.3704	0.0	0.0	0.3740	0.2557
0.0	0.3682	0.0	0.0	0.3491	0.2827
0.0	0.4475	0.0	0.0	0.4084	0.1440
0.0	0.3623	0.0	0.0	0.4933	0.1444
0.0	0.2493	0.0	0.0	0.4379	0.3128
0.0	0.1411	0.0	0.0	0.4037	0.4552
0.0	0.0560	0.0	0.0	0.3410	0.6030
0.0	0.1376	0.0	0.0	0.3805	0.4819

Optimal Weighting for the Year

0.0	0.3091	0.0	0.0	0.3697	0.3212
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Table 4-16. Monthly Performance for the Three-Site Network

Month	Network Power, Annual Weighting	Variance, Annual Weighting	Variance, Monthly Weightings	Variance, Monthly Weighting (Unconstrained)
1	0.593	0.212	0.168	0.168
2	0.944	0.418	0.389	0.389
3	1.317	0.574	0.554	0.554
4	1.089	0.675	0.662	0.662
5	1.170	0.449	0.442	0.442
6	1.314	0.469	0.462	0.462
7	1.233	0.539	0.485	0.485
8	0.933	0.330	0.298	0.298
9	0.907	0.446	0.436	0.436
10	0.838	0.470	0.439	0.439
11	0.885	0.428	0.355	0.355
12	0.779	0.356	0.311	0.311

Table 4-17. Monthly and Annual Normalized Weightings,
Two-Station Network

Optimal Weightings for each Month (unconstrained)

0.0	0.5192	0.0	0.0	0.4808	0.0
0.0	0.4869	0.0	0.0	0.5131	0.0
0.0	0.5920	0.0	0.0	0.4080	0.0
0.0	0.4718	0.0	0.0	0.5282	0.0
0.0	0.5404	0.0	0.0	0.4596	0.0
0.0	0.5405	0.0	0.0	0.4595	0.0
0.0	0.5251	0.0	0.0	0.4749	0.0
0.0	0.4496	0.0	0.0	0.5509	0.0
0.0	0.4496	0.0	0.0	0.5504	0.0
0.0	0.4558	0.0	0.0	0.5442	0.0
0.0	0.4347	0.0	0.0	0.5653	0.0
0.0	0.3821	0.0	0.0	0.6179	0.0

Constrained Weightings for each Month (non-negative)

0.0	0.5192	0.0	0.0	0.4808	0.0
0.0	0.4869	0.0	0.0	0.5131	0.0
0.0	0.5920	0.0	0.0	0.4080	0.0
0.0	0.4718	0.0	0.0	0.5282	0.0
0.0	0.5404	0.0	0.0	0.4596	0.0
0.0	0.5405	0.0	0.0	0.4595	0.0
0.0	0.5251	0.0	0.0	0.4749	0.0
0.0	0.4491	0.0	0.0	0.5509	0.0
0.0	0.4496	0.0	0.0	0.5504	0.0
0.0	0.4558	0.0	0.0	0.5442	0.0
0.0	0.4347	0.0	0.0	0.5653	0.0
0.0	0.3821	0.0	0.0	0.6179	0.0

Optimal Weighting for the Year

0.0	0.5107	0.0	0.0	0.4893	0.0
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Table 4-18. Monthly Performance for the Two-Site Network

Month	Network Power, Annual Weighting	Variance, Annual Weighting	Variance, Monthly Weightings	Variance, Monthly Weighting (Unconstrained)
1	0.572	0.363	0.363	0.363
2	1.058	0.774	0.773	0.773
3	1.391	0.695	0.678	0.678
4	1.102	0.704	0.700	0.700
5	1.220	0.558	0.556	0.556
6	1.463	0.670	0.667	0.667
7	1.226	0.507	0.507	0.507
8	1.006	0.373	0.362	0.362
9	0.803	0.441	0.433	0.433
10	0.678	0.478	0.473	0.473
11	0.757	0.547	0.533	0.533
12	0.735	0.557	0.524	0.524

Table 4-19. Monthly Performance for the One-Site Network

Month	Network Power	Variance
1	0.304	0.410
2	0.844	0.984
3	1.591	1.217
4	0.915	1.097
5	1.511	1.268
6	1.551	1.263
7	1.429	1.156
8	1.117	1.109
9	0.891	1.037
10	0.747	0.937
11	0.491	0.805
12	0.588	0.915

will cause an increase in the variance. An examination of the correlations of site four with the other sites, which are given in Table 4-8(f), shows that large correlations do indeed occur. There are correlations of 0.65 with site one, 0.58 with site five, and 0.45 with site six.

A comparison of the variances using annual weightings with the variances using monthly non-negative weightings shows how well the annual weights perform for any given month. If there is a great difference between such variances for a month (or months), it would be possible to recompute the annual weightings, using the monthly-weighting techniques described earlier, so as to force the resulting variances during those months to more closely approach the minimum values as shown by the variances for monthly weightings.

The six-site network using annual weightings performs reasonably well during most of the year. The greatest increase in the network variance over the monthly optimal variance is 30 percent which occurs for the first month. However, the network variance is quite small for that month anyway, which makes the large difference in variances relatively insignificant. The next greatest differences occur for months six, seven, and eleven for which the percentage increases over the optimal variance are 24, 22 and 25 percent, respectively.

The five-station network produces monthly variances using the annual weights that also follow a trend similar to that for the six-station network. Table 4-12 shows that the greatest differences between the variances for the monthly and annual weightings occur during months one,

six, seven and eleven. Month four has the greatest variance, 0.50, which is slightly above the variance for month four in the six-station network.

In the four-station network, however, the variances (Table 4-14) are significantly greater for some months than for the five-station network. For months three and four, the variances from the annual weighting are 27 and 18 percent greater, respectively, than the corresponding variances for the five-station network. An interesting event occurs in month six: The variance in the four-station network is actually 12 percent less than that for the five-station network. This result could be due to a large variance occurring in the sixth month for the station that was dropped when going from five to four stations. Fig. 4-1 shows the network groupings; station four was dropped to obtain the four-station network. Table 4-8(f) shows the variance for station four in the sixth month, which is 3.2×10^7 . Neither this value of the variance nor the mean is particularly large relative to the values for the other stations. However, station four is strongly correlated with stations one, five and six and stations five and six are both in the four and five-station networks. Removal of the strongly-correlated station four greatly reduces the variance, thereby creating the observed reversal of the normal trend as the number of stations in the network is decreased.

The trend of gradually increasing monthly variances continues with decreases to the three and two-station networks, as shown in Tables 4-16 and 4-18. Also there is less difference, in general, between the variances based on monthly and annual data. The two-station network in

particular shows very little difference between the two variances for each month.

Dropping from the two-station network to the single station (station two) produces a marked increase in the monthly variances as compared to the two-station network. The one-station variances are listed in Table 4-19. The large increase in the variances is simply due to removing the averaging effect that occurs when power from two stations is combined.

Two Station Network Evaluation

The two-station network that produced the smallest annual variance among all the possible two-station networks is composed of stations two and five (see Fig. 4-1). In the previous section is mentioned the lack of substantial difference in the variances based on monthly and annual data. This raises a question about two-station networks: Is the monthly variance resulting from annual weights always close to the monthly optimum? A number of possible two-station networks from the six available sites were examined, and the results are presented below.

The simplified three-station analysis developed in Chapter 2 shows that the greatest prospect for improvement by employing optimal weighting tends to occur for sites with different means and variances. The different variances for equal and optimal weighting also tends to be monotonic with the correlation coefficient, thus suggesting that for greatest improvement sites should exhibit correlation extremes.

An examination of the correlation matrix for the year, Table 4-2, and the annual variances, Table 4-1, shows some candidate two-station networks that might show a reduction in variance by using optimal in

place of equal weights. These are stations two and four, two and five, one and three, and one and two. Of course other two-station networks could also have been selected; an exhaustive search was not conducted.

Of the candidate networks listed above, the two-station network consisting of stations one and three produced as much improvement or more than the others. The monthly variances using equal weightings for each month are compared to the variances obtained by using optimal weightings for each month in Table 4-20.

Although the reduction in the variance is rather modest by employing equal rather than optimal gains, there is an improvement shown for several different months. Over a five percent reduction occurs for seven different months, with the greatest reduction being ten percent which occurs for month five.

The results show that actual wind data can have the necessary characteristics which allow optimal weights to reduce the variance resulting from equal weights in a two-station network. Although the reduction was not great for the example shown, other sites may show greater improvement. Also, it is anticipated that the greatest improvement from using optimal weighting will occur for networks composed of several, and not just two, stations.

Results for Equal and Optimal Weightings

A simplified approach to distributing wind generators among the sites in a wind generator network is to use equal weighting, with the same number at each site. In this section this approach is examined and compared with the network performance for optimal weightings. The

Table 4-20. Monthly Performance of Two-Station Network Using Sites 1 and 3

Month	Variance (equal weightings)	Variance (optimal weightings)
1	0.714	0.676
2	0.799	0.798
3	0.486	0.482
4	0.713	0.696
5	0.858	0.778
6	0.476	0.471
7	0.846	0.801
8	1.078	0.993
9	1.318	1.245
10	1.058	0.980
11	1.152	1.093
12	1.109	1.104

equal weightings are easily calculated as being proportional to the inverse of the sum of the mean wind powers from the network sites. The proportionality constant was chosen for convenience, to result in a value of one for the network mean power.

The network variances for equal and optimal weightings, with both sets of weightings chosen to produce the same average network power of one for each month, were computed for the different network configurations. The results are listed in Table 4-21. The monthly optimal variances in Table 4-21 differ from the corresponding variances in the previous tables since the monthly average network powers are not the same. By dividing the variance by the square of the monthly mean power, a normalized variance is obtained and that is the same for corresponding months for both sets of tables.

The results in Table 4-21 show that there can be a significant reduction in the network variance by using optimal, rather than equal, site weightings. This could result in considerable economic benefit in making a wind generator network feasible for large scale electrical power production. It should be noted, however, that not all months show a substantial difference between the variances for equal and optimal weightings. As shown in Chapter 2, the network variance is dependent on the means, variances and covariances among the sites, and there are some combinations of values that show little improvement in changing from equal to optimal weightings. Each proposed network must be analyzed to determine which allocation scheme to use.

An examination of the variances listed in Table 4-21 shows that

Table 4-21. Network Variances for Equal and Optimal Weights for Each Month

Month	6 Station Network		5 Station Network		4 Station Network		3 Station Network		2 Station Network		1 Station Network
	Var., Eq. Wts.	Var., Opt. Wts. ^a	Var.								
1	.416	.280	.424	.280	.371	.298	.588	.478	1.11	1.11	4.43
2	.342	.310	.353	.327	.394	.372	.460	.437	.691	.690	1.38
3	.179	.167	.179	.173	.238	.235	.327	.319	.362	.350	.481
4	.419	.390	.431	.409	.508	.497	.579	.558	.578	.576	1.31
5	.294	.266	.278	.268	.340	.314	.328	.323	.377	.374	.556
6	.274	.209	.266	.213	.217	.213	.270	.268	.314	.312	.525
7	.367	.288	.342	.288	.369	.317	.353	.319	.338	.337	.566
8	.326	.235	.301	.237	.375	.325	.395	.342	.365	.358	.889
9	.609	.492	.589	.492	.528	.494	.557	.531	.681	.672	1.31
10	.552	.491	.567	.504	.553	.513	.680	.625	1.04	1.03	1.68
11	.533	.377	.519	.377	.488	.387	.556	.453	.947	.930	3.33
12	.456	.370	.475	.374	.481	.404	.604	.514	1.02	.971	2.65

^aOptimal weights for each month are constrained to be non-negative.

there is little difference in the variances for equal and optimal weights for the two-station network, and there is only a very modest improvement shown for the three-station network. For the four-station network, however, there is a greater improvement by using optimal weights. The five and six station networks show substantial reduction in the optimal-weight variance from the equal-weight variance for some months. There are seven months for the five-station network (months one, six, seven, eight, nine, eleven, twelve) that show at least a 15 percent reduction in the variance, with the greatest being 34 percent for the first month. In the six station network the smallest improvement gained by using the optimal monthly weightings is seven percent, which occurs for months three and four, while five different months (one, six, seven, eight, and eleven) show a variance reduction of greater than 20 percent and two more months have a reduction of 19 percent.

Of course, it would not be possible to reallocate actual wind generators among various sites on a month-to-month basis, but the monthly analysis does show an advantage that optimal weighting can have over equal weighting. Another feature of the monthly analysis is that it could suggest a plan for scheduled maintenance of the wind generators at sites that are contributing the most to the network variance. During subsequent months (or seasons), wind generators at other sites would be scheduled for maintenance when their removal from service would contribute the most to reduced network variance. In addition, if less than maximum power from the network is required for periods of time, the same approach could be followed for determining which generators or sites to shut down.

Average Network Power

The average actual network power produced from each combination of sites can be determined by applying the normalized weightings to the mean power density values given in Table 3-1, and then including the effect of the efficiencies given in eqs. 3-3, 3-4, and 3-5. In previous sections within this chapter, the constraint on the weightings was chosen so as to produce unity average network power. This allows a direct comparison of items such as power duration curves for different network configurations. The average network power presented below gives the power produced per square meter of swept windmill area, so that for a given number and size of wind generators, the total power produced can be directly calculated.

The normalized optimal weighting for the two-station network is, from Table 4-17, 0.5107 applied to station 2 and 0.4893 applied to station 5. Using eq. 3-7 (which includes the efficiency factors) and the means of wind speeds cubed for stations 2 and 5 from Table 3-1, the average network power per square foot of swept area for the two station network is 11.1 watts/ft² of 119 watts/m². Similarly, the average network power densities for all of the network combinations can be found; these are listed in Table 4-22.

The greatest power density is for the single-station configuration since that station (site 2) has the greatest average wind speeds. Adding other stations to the network can then only reduce the power density, since all other sites have lower average wind speeds.

As an example of power production from a large wind generator

Table 4-22. Network Power Densities

Number of Stations in Network	Average Network Power Density
1	137 w/m ²
2	119 w/m ²
3	115 w/m ²
4	115 w/m ²
5	113 w/m ²
6	111 w/m ²

network, suppose that 1000 wind generators are to be distributed among the four most promising sites. If each wind generator sweeps a circular area 200 ft in diameter, then the swept area is 2919 m² for each windmill. The total average power can now be found for this four-station network.

$$\begin{aligned} \text{average power} &= 1000 \text{ units} \times 2919 \text{ m}^2/\text{units} \times 115 \text{ w/m}^2 \\ &= 336 \text{ megawatts} \end{aligned}$$

The distribution of the 1000 units among the four sites is according to the normalized weights given in Table 4-13. This yields 227 units at site 2, 211 at site 3, 306 at site 5, and 256 wind generators at site 6. If all 1000 units were placed at the single best site, which is site 2, the total average power would increase to 400 megawatts. However, this increase in average power would be accompanied by an increase in the variance of the power fluctuations by a factor of 3.8, or nearly double the root-mean-square fluctuations. Whether this increase in network

power fluctuations is significant depends on a particular utility's circumstances. The rms fluctuations are calculated in a manner similar to the average power for the network; the results are rms values of 207 and 404 megawatts for the networks composed of four and one stations, respectively.

CHAPTER 5.

CONCLUSIONS AND RECOMMENDATIONS

The electrical power that can be produced by large scale wind generators is significant and wind generator "farms" are being planned. A network of wind generator farms could be planned and coordinated so as to maximize the usefulness of the electrical power, including selection of most desirable sites and minimizing the deviations of the network power from desired levels.

Combining the generated power from various wind generator sites may be best accomplished through placement of unequal numbers of wind generators at the various sites. This approach can reduce the fluctuations in generated power from the network, while maintaining significant power production. If the network sites have similar means and variances of wind speeds, then equal numbers at the different sites are best. However, if sites with sufficiently different means and/or variances are to be used in a wind generator network, then unequal allocations of windmills at the various sites are best. The actual proportion of number of wind generators at each location depends on site-to-site correlations as well as the wind statistics at each site.

The installation of only a few wind generators as part of an electric utility's generating capacity will be beneficial for the energy production, while the fluctuations in power from the wind generators will be insignificant relative to normal load variations. Thus, an initial site should be selected that maximizes the useful energy in the wind. After an initial site has been developed with the installation of

wind generators, additional sites should be selected based on power production in coordination with the first sites. As an increasing proportion of electrical power is produced from the wind, more and more emphasis must be given to planning and operation of the wind generator network.

A wind generator network can have a smoothing effect on generated power through geographical dispersion as well as by optimal allocation of wind generators among the various sites. Both of these effects have been examined in this study and both can contribute significantly to smoothing the power fluctuations. There is a trend of diminishing return as more sites are added to a network. For the six sites considered in this study, it appears that a network of the four best sites may be sufficient, even if all six sites are available for development. The cost of developing an additional site, beyond an already adequate number of developed sites, may not be justified from the view of contributing toward smoother network power. Of course, an additional site might be developed simply on the basis of the value of the electrical energy at that location. By geographical dispersion of wind generators and optimal allocations among the network sites, some firm generating capacity from the wind generator network can be possible. A proper assessment of the amount of firm capacity would include expected maintenance schedules and expected failure rates as well as the wind statistics.

Suggested future work includes the development of adequate wind power forecasting techniques and utility operational methods for large scale wind generator networks. These are discussed briefly below.

Short term forecasting of wind power is needed in order to use the

wind power generators most effectively, since conventional generating facilities require time to adjust to changes in scheduled power production. Working with a fluctuating power source, such as the wind, always involves a degree of uncertainty. It is anticipated that operating procedures can be developed that will overcome most of this problem. It is necessary that records be kept of the wind power history at each site so that the best possible forecasting can be achieved to determine the maintenance and operational schedules. Seasonal trends involving several years of data would be needed for the forecasts used in determining the maintenance schedule, while the operational schedule would likely involve less data for producing the short-term wind power forecast.

Although wind power uncertainties and fluctuations may be new to the electrical power industry, other fluctuating power sources are dealt with routinely. Hydroelectric systems are a prime example of success with a fluctuating power source (of course, the time scale of fluctuations is vastly different between hydroelectric and wind power systems). Another fluctuating electric power source is becoming more apparent in these times of inflation and limited supply, and that is simply oil itself! The uncertainties in wind power may eventually seem insignificant, indeed, if costs of conventional fuel supplies become excessive.

APPENDIX

Wind Speed Plots

Actual wind data used in this study consist of hourly averages for a year from each of the six sites identified in Chapter 3. Thus, there are 8,760 (the number of hours in a year) data points for each station. A listing or plot of such a large number of values would be cumbersome at best, and would not be of any great benefit. In order to view trends and levels in the data, moving averages over 24 hours were computed. Plots of the resulting smoothed data for each site are presented in Figs. A-1 through A-6. Also shown are the portions of time when data are missing, which were caused by malfunctions in the data collection process. Wind speeds in the range of roughly 20 to 40 mph are desirable for wind generator use; the figures show that the 24 hour averages are within that range for significant portions of time.

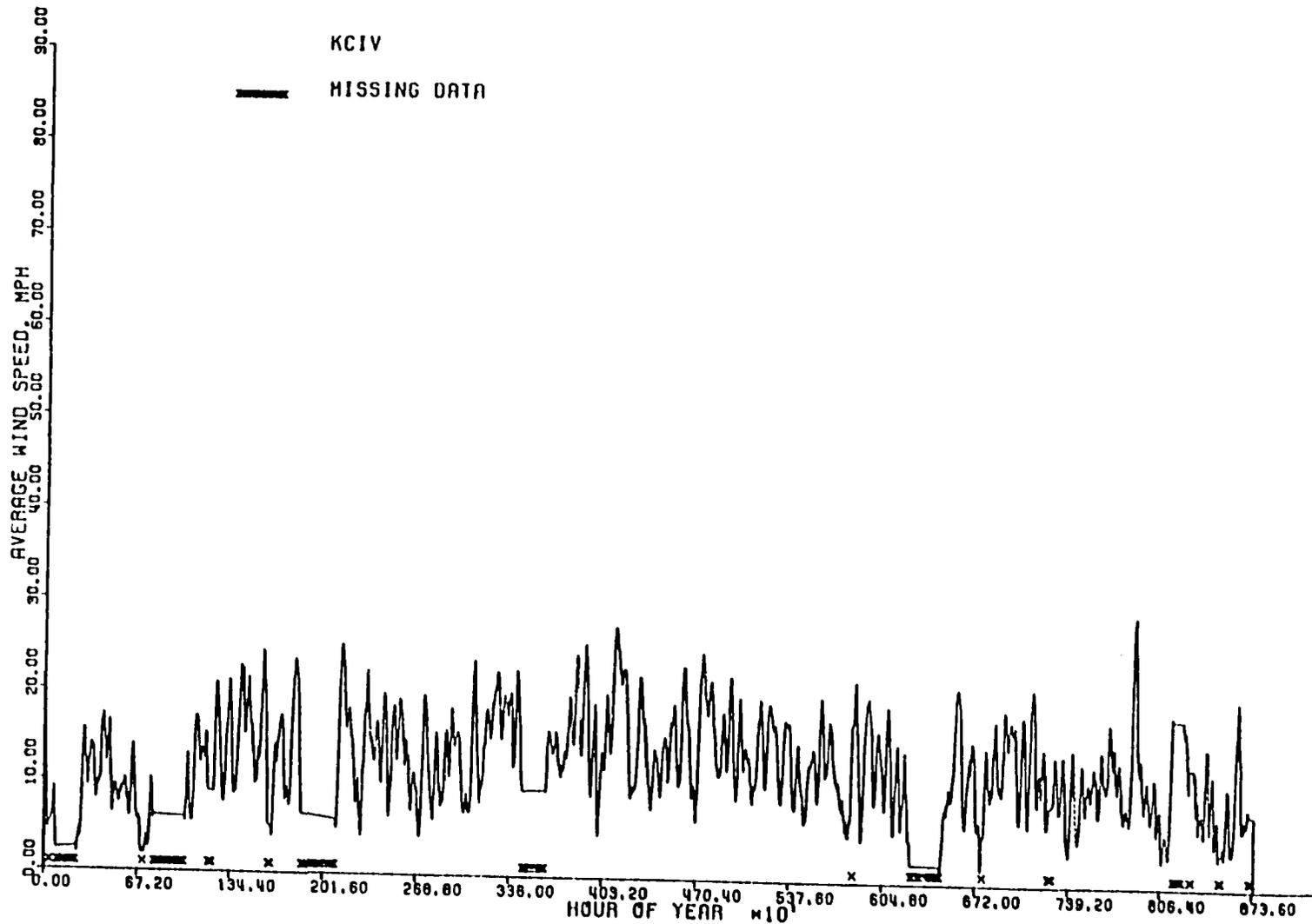


Fig. A-1. Wind Speed for Station 1, 24 Hour Average

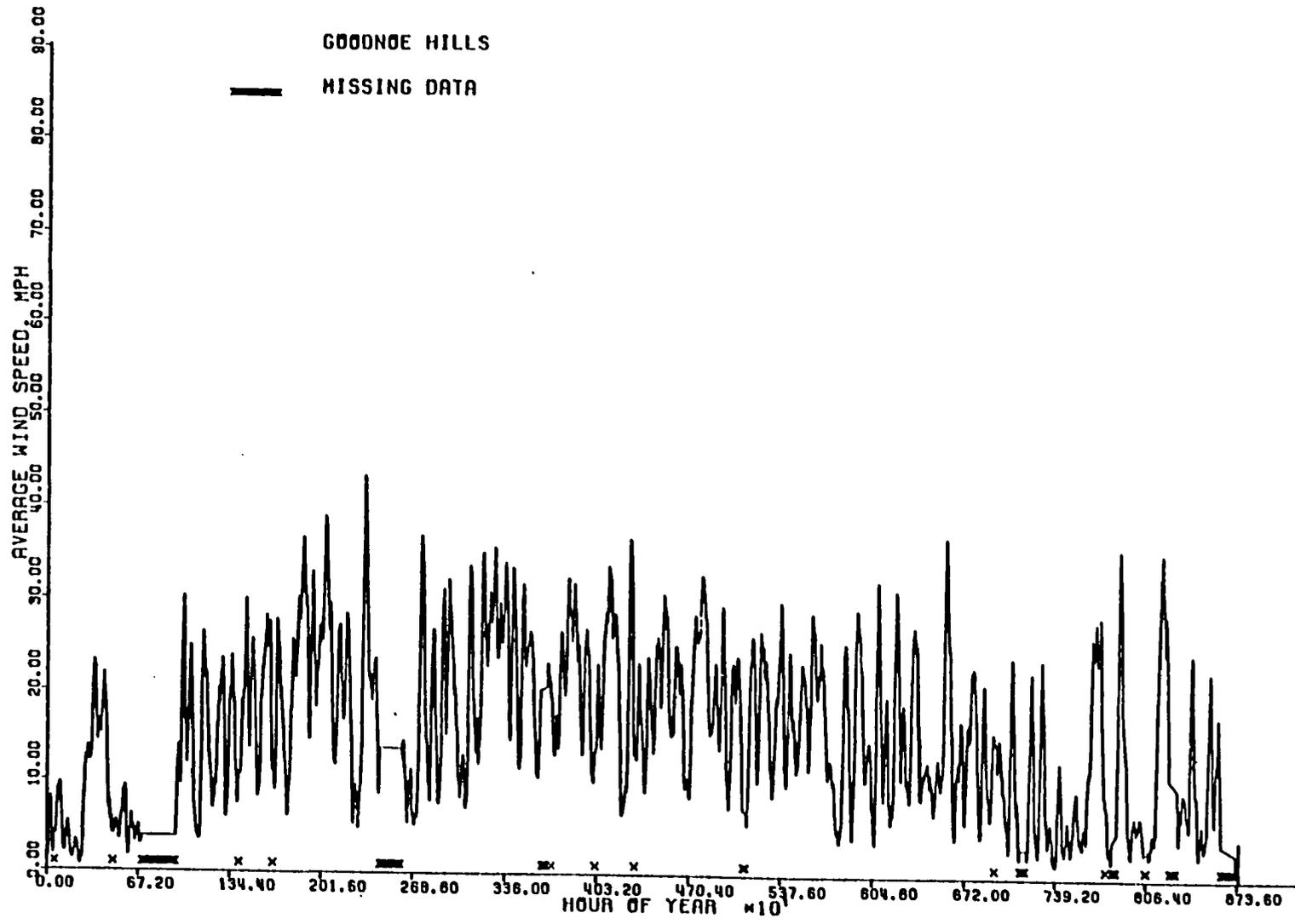


Fig. A-2. Wind Speed for Station 2, 24 Hour Average

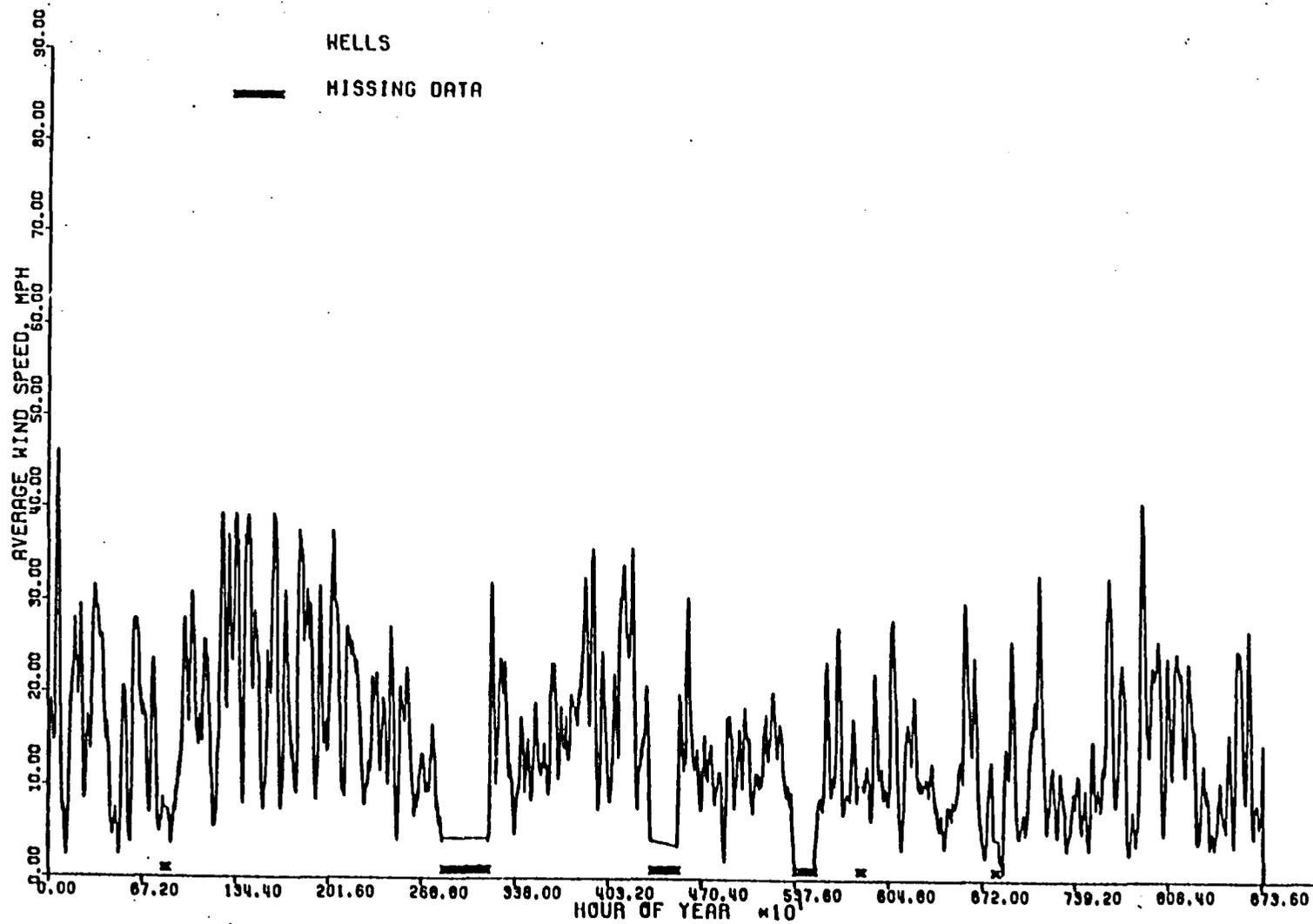


Fig. A-3. Wind Speed for Station 3, 24 Hour Average

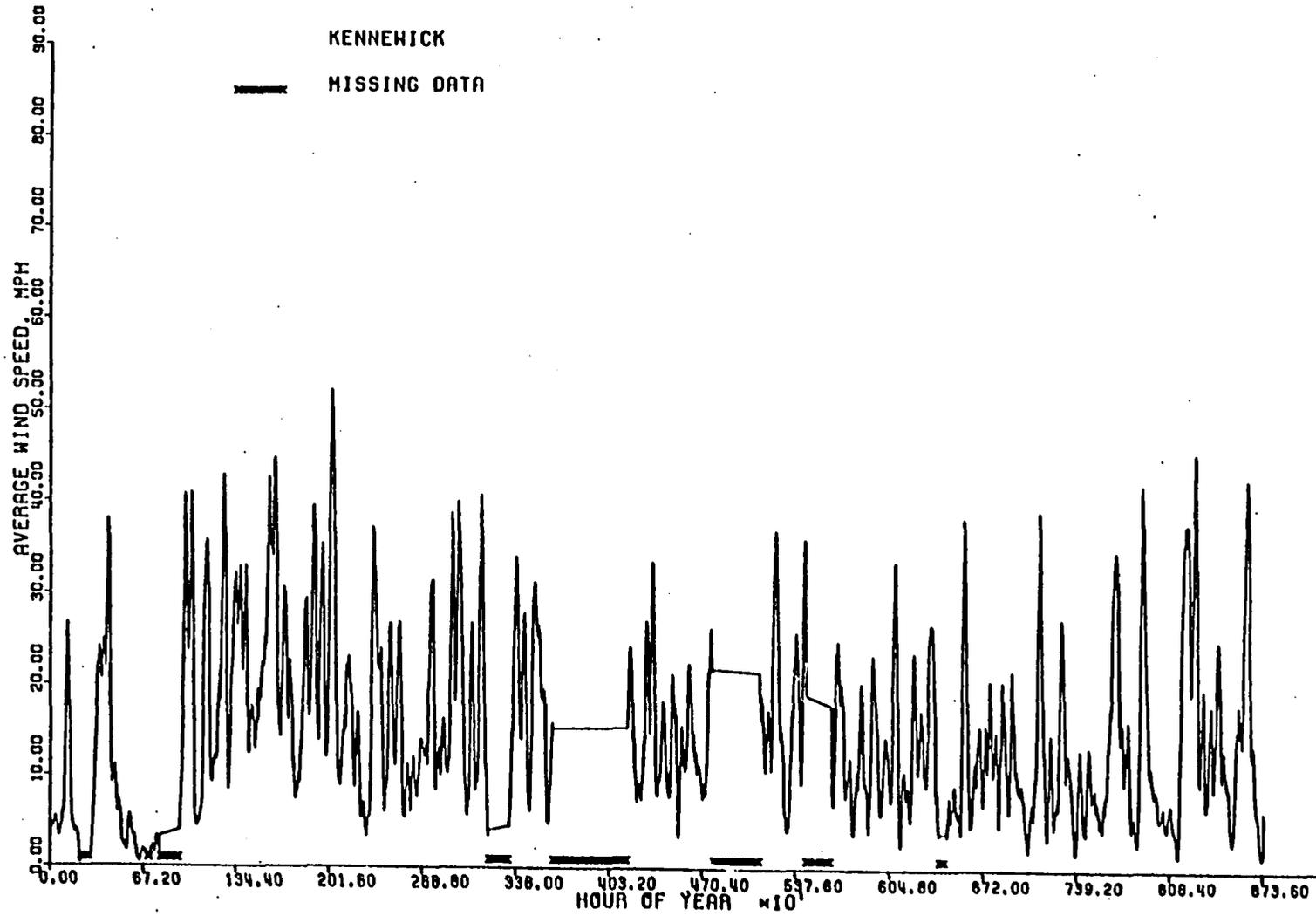


Fig. A-4. Wind Speed for Station 4, 24 Hour Average

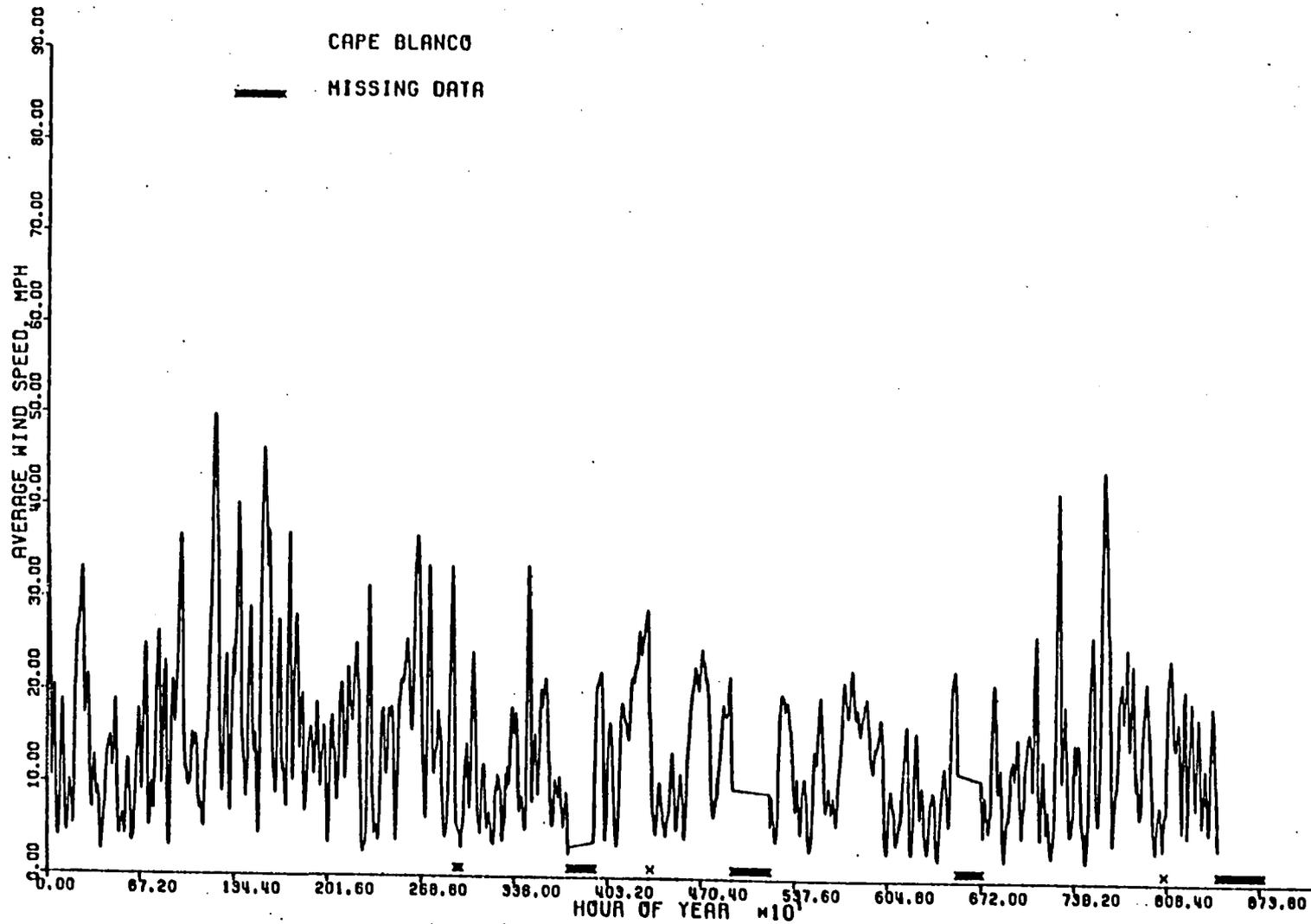


Fig. A-5. Wind Speed for Station 5, 24 Hour Average

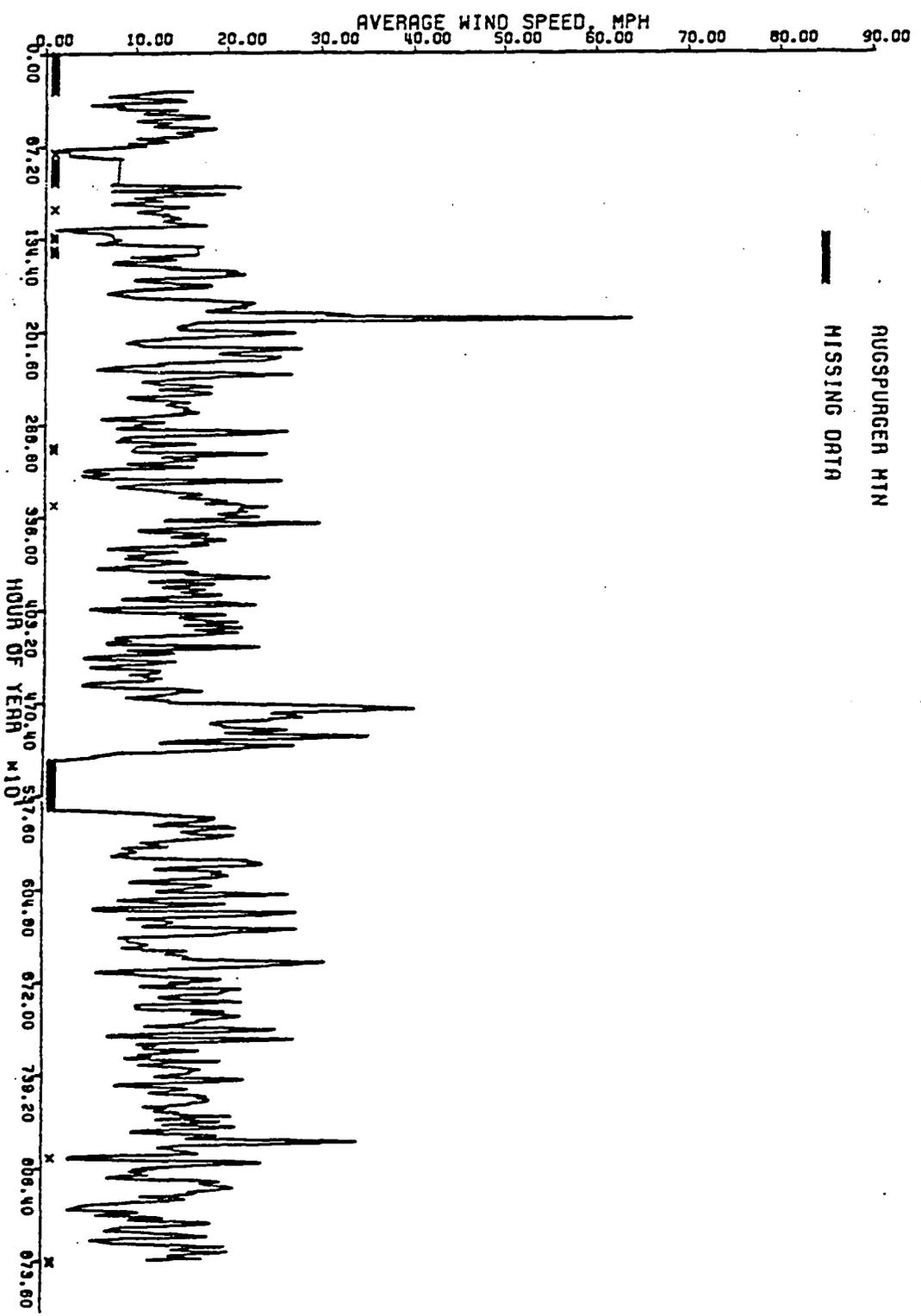


Fig. A-6. Wind Speed for Station 6, 24 Hour Average

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